Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "5 Inverse trig functions\5.6 Inverse cosecant"

Test results for the 178 problems in "5.6.1 u (a+b arccsc(c x))^n.m"

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a+b\,\text{ArcCsc}[c\,x]\right)^3 \, dx$$

Optimal (type 4, 220 leaves, 11 steps):

$$\frac{b^2 x \left(a+b\,\text{ArcCsc}[c\,x]\right)}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2 x^2}}\; x^2 \left(a+b\,\text{ArcCsc}[c\,x]\right)^2}{2\,c}+\frac{1}{3} x^3 \left(a+b\,\text{ArcCsc}[c\,x]\right)^3+$$
$$\frac{b\left(a+b\,\text{ArcCsc}[c\,x]\right)^2 \text{ArcTanh}\left[e^{i\,\text{ArcCsc}[c\,x]}\right]}{c^3}+\frac{b^3\,\text{ArcTanh}\left[\sqrt{1-\frac{1}{c^2 x^2}}\right]}{c^3}-\frac{i\,b^2\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{PolyLog}\left[2,-e^{i\,\text{ArcCsc}[c\,x]}\right]}{c^3}+$$
$$\frac{i\,b^2\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{PolyLog}\left[2,e^{i\,\text{ArcCsc}[c\,x]}\right]}{c^3}+\frac{b^3\,\text{PolyLog}\left[3,-e^{i\,\text{ArcCsc}[c\,x]}\right]}{c^3}-\frac{b^3\,\text{PolyLog}\left[3,e^{i\,\text{ArcCsc}[c\,x]}\right]}{c^3}$$

Result (type 4, 580 leaves):

$$\frac{a^3 x^3}{3} + \frac{a^2 b x^2 \sqrt{\frac{-1+c^2 x^2}{c^2 x^2}}}{2 c} + a^2 b x^3 \text{ArcCsc}[c x] + \frac{a^2 b \text{Log}\left[x \left(1 + \sqrt{\frac{-1+c^2 x^2}{c^2 x^2}}\right)\right]}{2 c^3} +$$

$$\frac{1}{8 c^3} a b^2 \left(-8 i \text{PolyLog}\left[2, -e^{i \text{ArcCsc}[c x]}\right] + 2 c^3 x^3 \left(2 + 4 \text{ArcCsc}[c x]^2 - 2 \text{Cos}[2 \text{ArcCsc}[c x]] - \frac{3 \text{ArcCsc}[c x] \text{Log}\left[1 - e^{i \text{ArcCsc}[c x]}\right]}{c x} + \frac{3 \text{ArcCsc}[c x] \text{Log}\left[1 + e^{i \text{ArcCsc}[c x]}\right]}{c x} + \frac{4 i \text{PolyLog}\left[2, e^{i \text{ArcCsc}[c x]}\right]}{c^3 x^3} + 2 \text{ArcCsc}[c x] \text{Sin}[2 \text{ArcCsc}[c x]] + \text{ArcCsc}[c x] \text{Log}\left[1 - e^{i \text{ArcCsc}[c x]}\right] \text{Sin}[3 \text{ArcCsc}[c x]] - \text{ArcCsc}[c x] \text{Log}\left[1 + e^{i \text{ArcCsc}[c x]}\right] \text{Sin}[3 \text{ArcCsc}[c x]]\right)\right) +$$

$$\frac{1}{48 c^3} b^3 \left(24 \text{ArcCsc}[c x] \text{Cot}\left[\frac{1}{2} \text{ArcCsc}[c x]\right] + 4 \text{ArcCsc}[c x]^3 \text{Cot}\left[\frac{1}{2} \text{ArcCsc}[c x]\right] + 6 \text{ArcCsc}[c x]^2 \text{Csc}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^2 + \frac{\text{ArcCsc}[c x]^3 \text{Csc}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^4}{c x} - 24 \text{ArcCsc}[c x]^2 \text{Log}\left[1 - e^{i \text{ArcCsc}[c x]}\right] + 24 \text{ArcCsc}[c x]^2 \text{Log}\left[1 + e^{i \text{ArcCsc}[c x]}\right] - 48 \text{Log}\left[\text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right] - 48 i \text{ArcCsc}[c x] \text{PolyLog}\left[2, -e^{i \text{ArcCsc}[c x]}\right] + 48 i \text{ArcCsc}[c x] \text{PolyLog}\left[2, e^{i \text{ArcCsc}[c x]}\right] + 48 \text{PolyLog}\left[3, -e^{i \text{ArcCsc}[c x]}\right] - 48 \text{PolyLog}\left[3, e^{i \text{ArcCsc}[c x]}\right] - 6 \text{ArcCsc}[c x]^2 \text{Sec}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^2 + 16 c^3 x^3 \text{ArcCsc}[c x]^3 \text{Sin}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]^4 + 24 \text{ArcCsc}[c x] \text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right] + 4 \text{ArcCsc}[c x]^3 \text{Tan}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right)$$

Problem 51: Unable to integrate problem.

$$\int x^2 \sqrt{d + e x} \, (a + b \text{ArcCsc}[c x]) \, dx$$

Optimal (type 4, 496 leaves, 31 steps) :

$$\frac{4\,b\,d\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{105\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}-\frac{4\,b\,(d+e\,x)^{3/2}\,\left(1-c^2\,x^2\right)}{35\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,(d+e\,x)^{3/2}\,(a+b\,\mathrm{ArcCsc}[c\,x])}{3\,e^3}-\frac{4\,d\,(d+e\,x)^{5/2}\,(a+b\,\mathrm{ArcCsc}[c\,x])}{5\,e^3}+$$

$$\frac{2\,(d+e\,x)^{7/2}\,(a+b\,\mathrm{ArcCsc}[c\,x])}{7\,e^3}+\frac{4\,b\,\left(5\,c^2\,d^2-9\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{105\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-$$

$$\frac{4\,b\,d\,\left(9\,c^2\,d^2-e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{105\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{32\,b\,d^4\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\mathrm{EllipticPi}\left[2,\mathrm{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{105\,c\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 9, 870 leaves):

$$-\frac{a\,d^3\,\sqrt{d+e\,x}\,\mathrm{Beta}\left[-\frac{e\,x}{d},3,\frac{3}{2}\right]}{e^3\,\sqrt{1+\frac{e\,x}{d}}}+\frac{1}{c^4}$$

$$b\left(-\frac{1}{\sqrt{d+e\,x}}c\left(e+\frac{d}{x}\right)x\left(-\frac{4\left(-5\,c^2\,d^2+9\,e^2\right)\sqrt{1-\frac{1}{c^2\,x^2}}}{105\,e^2}-\frac{16\,c^3\,d^3\,\mathrm{ArcCsc}[c\,x]}{105\,e^3}-\frac{2}{7}\,c^3\,x^3\,\mathrm{ArcCsc}[c\,x]-\frac{2\,c^2\,x^2\left(2\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}+c\,d\,\mathrm{ArcCsc}[c\,x]\right)}{35\,e}-\right.\right.$$

$$\left.\frac{8\,c\,x\left(c\,d\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}-c^2\,d^2\,\mathrm{ArcCsc}[c\,x]\right)}{105\,e^2}\right)-$$

$$\frac{1}{105\,e^3\,\sqrt{d+e\,x}}\,2\,\sqrt{e+\frac{d}{x}}\,\sqrt{c\,x}\left(\frac{2\left(9\,c^3\,d^3\,e-c\,d\,e^3\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\right.$$

$$\frac{2\left(8 c^4 d^4+5 c^2 d^2 e^2-9 e^4\right) \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2} \text{ EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c x)^{3/2}}+\frac{1}{c d \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} \sqrt{c x} \left(-2+c^2 x^2\right)}$$

$$2\left(-5 c^3 d^3 e+9 c d e^3\right) \text{Cos}[2 \text{ArcCsc}[c x]] \left((c d+c e x)\left(-1+c^2 x^2\right)+c^2 d x \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2} \text{ EllipticF}\left[\text{ArcSin}\left[\right.\right.\right.$$

$$\left.\left.\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]-\frac{1}{\sqrt{\frac{e (1+c x)}{-c d+e}}} c x (1+c x) \sqrt{\frac{e-c e x}{c d+e}} \sqrt{\frac{c d+c e x}{c d-e}} \left((c d+e) \text{ EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c d+c e x}{c d-e}}\right], \frac{c d-e}{c d+e}\right]-\right.$$

$$\left.\left.\left.e \text{ EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c d+c e x}{c d-e}}\right], \frac{c d-e}{c d+e}\right]\right)+c e x \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2} \text{ EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]\right)\right)\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int x \sqrt{d+e x} (a+b \text{ ArcCsc}[c x]) \, dx$$

Optimal (type 4, 404 leaves, 24 steps) :

$$-\frac{4 b \sqrt{d+e x} \left(1-c^2 x^2\right)}{15 c^3 \sqrt{1-\frac{1}{c^2 x^2}} x}-\frac{2 d (d+e x)^{3/2} (a+b \text{ArcCsc}[c x])}{3 e^2}+$$

$$\frac{2 (d+e x)^{5/2} (a+b \text{ArcCsc}[c x])}{5 e^2}-\frac{8 b d \sqrt{d+e x} \sqrt{1-c^2 x^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{15 c^2 e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{\frac{c (d+e x)}{c d+e}}}+$$

$$\frac{4 b \left(3 c^2 d^2-e^2\right) \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{15 c^4 e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}+\frac{8 b d^3 \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right],\frac{2 e}{c d+e}\right]}{15 c e^2 \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 368 leaves):

$$\frac{1}{15}\left(\frac{4 b \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}{c}+\frac{2 a \sqrt{d+e x} \left(-2 d^2+d e x+3 e^2 x^2\right)}{e^2}+\right.$$

$$\frac{2 b \sqrt{d+e x} \left(-2 d^2+d e x+3 e^2 x^2\right) \text{ArcCsc}[c x]}{e^2}-\left(4 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}}\right.$$

$$\left(-2 c d (c d-e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]+\left(-c^2 d^2-2 c d e+e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\right.\right.$$

$$\left.\left.\left.\frac{c d+e}{c d-e}\right]+2 c^2 d^2 \text{EllipticPi}\left[1+\frac{e}{c d},i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right],\frac{c d+e}{c d-e}\right]\right)\right)\Big/\left(c^3 e^2 \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}} x\right)\right)$$

Problem 53: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d+e x} (a+b \text{ArcCsc}[c x]) \, dx$$

Optimal (type 4, 315 leaves, 15 steps):

$$\frac{2\,(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e}-\frac{4\,b\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-$$

$$\frac{4\,b\,d\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{4\,b\,d^2\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 701 leaves) :

$$\frac{2a\,(d+e\,x)^{3/2}}{3\,e}+\frac{1}{c^2}\,b\left(-\frac{(c\,d+c\,e\,x)\left(-\frac{4}{3}\sqrt{1-\frac{1}{c^2x^2}}-\frac{2\,c\,d\,\text{ArcCsc}[c\,x]}{3\,e}-\frac{2}{3}\,c\,x\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x}}-\right.$$

$$\frac{1}{3\,e\sqrt{e+\frac{d}{x}}\,\sqrt{c\,x}\,\sqrt{d+e\,x}}\,2\,(c\,d+c\,e\,x)\left(\frac{2\,c\,d\,e\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\right.$$

$$\frac{2\,(c^2d^2-e^2)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\left(2\,e\,\text{Cos}[2\,\text{ArcCsc}[c\,x]]\right.$$

$$\left((c\,d+c\,e\,x)\,(-1+c^2x^2)+c^2\,d\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]-\frac{1}{\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}}c\,x\,(1+c\,x)\right.$$

$$\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\left((c\,d+e)\,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]-e\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]\right)+$$

$$\left.\left.\left.c\,e\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\right)\Big/\left(\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{e+\frac{d}{x}}\,\sqrt{c\,x}\,(-2+c^2x^2)\right)\right)\right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e\,x)^{3/2}\,(a+b\,\text{ArcCsc}[c\,x])\,dx$$

Optimal (type 4, 372 leaves, 22 steps):

$$-\frac{4\,b\,e\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{15\,c^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,(d+e\,x)^{5/2}\,(a+b\,\text{ArcCsc}[c\,x])}{5\,e}-\frac{28\,b\,d\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-$$

$$\frac{4\,b\,\left(2\,c^2\,d^2+e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^4\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{4\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{5\,c\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 333 leaves) :

$$\frac{1}{15}\left(\frac{4\,b\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}{c}+\frac{6\,a\,(d+e\,x)^{5/2}}{e}+\frac{6\,b\,(d+e\,x)^{5/2}\,\text{ArcCsc}[c\,x]}{e}-\right.$$

$$\left(4\,i\,b\,\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\left(-7\,c\,d\,(c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+\right.\right.$$

$$\left(9\,c^2\,d^2-7\,c\,d\,e+e^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-$$

$$\left.\left.\left.3\,c^2\,d^2\,\text{EllipticPi}\left[1+\frac{e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)\Big/\left(c^3\,e\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 57: Unable to integrate problem.

$$\int\frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 714 leaves, 27 steps) :

$$-\frac{4\,b\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{35\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}}+\frac{4\,b\,d\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{21\,c^3\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}-\frac{2\,d^3\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4}+\frac{2\,d^2\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4}-$$

$$\frac{6\,d\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^4}+\frac{2\,\left(d+e\,x\right)^{7/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{7\,e^4}-\frac{24\,b\,d^2\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{35\,c^2\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+$$

$$\frac{4\,b\,\left(2\,c^2\,d^2-9\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{105\,c^4\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+\frac{64\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{35\,c^2\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{32\,b\,d\,\left(c\,d-e\right)\,\left(c\,d+e\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{105\,c^4\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{64\,b\,d^4\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{35\,c\,e^4\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 9, 873 leaves) :

$$\frac{a\,d^4\,\sqrt{1+\frac{e\,x}{d}}\,\text{Beta}\left[-\frac{e\,x}{d},4,\frac{1}{2}\right]}{e^4\,\sqrt{d+e\,x}}+$$

$$\frac{1}{c^4}\,b\,\left(-\frac{1}{\sqrt{d+e\,x}}c\,\left(e+\frac{d}{x}\right)\,x\,\left(-\frac{4\,\left(16\,c^2\,d^2+9\,e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}}{105\,e^3}+\frac{32\,c^3\,d^3\,\text{ArcCsc}[c\,x]}{35\,e^4}-\frac{2\,c^3\,x^3\,\text{ArcCsc}[c\,x]}{7\,e}-\frac{4\,c^2\,x^2\,\left(e\,\sqrt{1-\frac{1}{c^2\,x^2}}-3\,c\,d\,\text{ArcCsc}[c\,x]\right)}{35\,e^2}+\right.\right.$$

$$\frac{4\,c\,x\left(5\,c\,d\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}-12\,c^2\,d^2\,\text{ArcCsc}[c\,x]\right)}{105\,e^3}\Bigg)+$$

$$\frac{1}{105\,e^4\,\sqrt{d+e\,x}}\,2\,\sqrt{e+\frac{d}{x}}\,\sqrt{c\,x}\,\Bigg(\frac{2\,\left(40\,c^3\,d^3\,e+8\,c\,d\,e^3\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+$$

$$\frac{2\,\left(48\,c^4\,d^4+16\,c^2\,d^2\,e^2+9\,e^4\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\frac{1}{c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\sqrt{e+\frac{d}{x}}\,\sqrt{c\,x}\,\left(-2+c^2\,x^2\right)}$$

$$2\left(-16\,c^3\,d^3\,e-9\,c\,d\,e^3\right)\text{Cos}[2\,\text{ArcCsc}[c\,x]]\,\Bigg(\left(c\,d+c\,e\,x\right)\left(-1+c^2\,x^2\right)+c^2\,d\,x\,\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\Big[\text{ArcSin}\Big[$$

$$\frac{\sqrt{1-c\,x}}{\sqrt{2}}\Big],\frac{2\,e}{c\,d+e}\Big]-\frac{1}{\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}}c\,x\,(1+c\,x)\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\,\Bigg(\left(c\,d+e\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]-$$

$$e\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]\Bigg)+c\,e\,x\,\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\Bigg)\Bigg)\Bigg)$$

Problem 58: Unable to integrate problem.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 530 leaves, 20 steps):

$$-\frac{4\,b\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{15\,c^3\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^3}-\frac{4\,d\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^3}+\frac{2\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^3}+$$

$$\frac{4\,b\,d\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{5\,c^2\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-\frac{32\,b\,d^2\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^2\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,\left(c\,d-e\right)\,\left(c\,d+e\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^4\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{32\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 9, 784 leaves) :

$$-\frac{a\,d^3\sqrt{1+\frac{e\,x}{d}}\ \text{Beta}\left[-\frac{e\,x}{d},3,\frac{1}{2}\right]}{e^3\sqrt{d+e\,x}}+\frac{1}{c^3}\,b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4\,c\,d\sqrt{1-\frac{1}{c^2x^2}}}{5\,e^2}-\frac{16\,c^2d^2\,\text{ArcCsc}[c\,x]}{15\,e^3}-\frac{2\,c^2x^2\,\text{ArcCsc}[c\,x]}{5\,e}-\frac{4\,c\,x\left(e\sqrt{1-\frac{1}{c^2x^2}}-2\,c\,d\,\text{ArcCsc}[c\,x]\right)}{15\,e^2}\right)}{\sqrt{d+e\,x}}-\right.$$

$$\frac{1}{15\,e^3\sqrt{d+e\,x}}\,2\sqrt{e+\frac{d}{x}}\sqrt{c\,x}\left(\frac{2\left(7\,c^2d^2e+e^3\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\right.$$

$$\frac{2\left(8\,c^3d^3+3\,c\,d\,e^2\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\ \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}-\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\sqrt{c\,x}\left(-2+c^2x^2\right)}$$

$$6\,c\,d\,e\,\text{Cos}[2\,\text{ArcCsc}[c\,x]]\left(\left(c\,d+c\,e\,x\right)\left(-1+c^2x^2\right)+c^2d\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]-\right.$$

$$\frac{1}{\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}}c\,x\,(1+c\,x)\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\left(\left(c\,d+e\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]-\right.$$

$$\left.\left.\left.\left.e\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]\right)+c\,e\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\ \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\right)\right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 344 leaves, 14 steps):

$$-\frac{2\,d\,\sqrt{d+e\,x}\,(a+b\,\text{ArcCsc}[c\,x])}{e^2}+\frac{2\,(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e^2}-\frac{4\,b\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+$$

$$\frac{8\,b\,d\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}+\frac{8\,b\,d^2\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 289 leaves):

$$\frac{1}{3\,e^2}2\left(a\,(-2\,d+e\,x)\,\sqrt{d+e\,x}+b\,(-2\,d+e\,x)\,\sqrt{d+e\,x}\,\text{ArcCsc}[c\,x]+\frac{1}{c^2\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}\right.$$

$$2\,i\,b\,\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\,\left((c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+\right.$$

$$\left.\left.(c\,d+e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-2\,c\,d\,\text{EllipticPi}\left[1+\frac{e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)\right)$$

Problem 63: Unable to integrate problem.

$$\int \frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x)^{3/2}}\,dx$$

Optimal (type 4, 551 leaves, 23 steps):

$$-\frac{4\,b\,\sqrt{d+e\,x}\,\left(1-c^2\,x^2\right)}{15\,c^3\,e^2\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^3\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4\,\sqrt{d+e\,x}}+$$

$$\frac{6\,d^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4}-\frac{2\,d\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4}+\frac{2\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{5\,e^4}+$$

$$\frac{32\,b\,d\,\sqrt{d+e\,x}\,\sqrt{1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^2\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-\frac{8\,b\,d^2\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{c^2\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,\left(2\,c^2\,d^2+e^2\right)\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{15\,c^4\,e^3\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}-\frac{64\,b\,d^3\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{5\,c\,e^4\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 9, 814 leaves) :

$$\frac{a\, d^4 \left(1+\frac{e x}{d}\right)^{3/2} \text{Beta}\left[-\frac{e x}{d}, 4, -\frac{1}{2}\right]}{e^4 (d+e x)^{3/2}} + \frac{1}{c^4} b \left(-\frac{1}{(d+e x)^{3/2}} c^2 \left(e+\frac{d}{x}\right)^2 x^2\right.$$

$$\left(\frac{32 c d \sqrt{1-\frac{1}{c^2 x^2}}}{15 e^3} - \frac{32 c^2 d^2 \text{ArcCsc}[c x]}{5 e^4} + \frac{2 c^2 d^2 \text{ArcCsc}[c x]}{e^3 \left(e+\frac{d}{x}\right)} - \frac{2 c^2 x^2 \text{ArcCsc}[c x]}{5 e^2} - \frac{2 c x \left(2 e \sqrt{1-\frac{1}{c^2 x^2}} - 9 c d \text{ArcCsc}[c x]\right)}{15 e^3}\right) -$$

$$\frac{1}{15 e^4 (d+e x)^{3/2}} 2 \left(e+\frac{d}{x}\right)^{3/2} (c x)^{3/2} \left(\frac{2 \left(32 c^2 d^2 e + e^3\right) \sqrt{\frac{c d + c e x}{c d+e}} \sqrt{1-c^2 x^2} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c x)^{3/2}} +\right.$$

$$\frac{2 \left(48 c^3 d^3 + 8 c d e^2\right) \sqrt{\frac{c d + c e x}{c d+e}} \sqrt{1-c^2 x^2} \text{ EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c x)^{3/2}} - \left(16 c d e \text{ Cos}[2 \text{ArcCsc}[c x]]\right.$$

$$\left((c d + c e x) \left(-1+c^2 x^2\right) + c^2 d x \sqrt{\frac{c d + c e x}{c d + e}} \sqrt{1-c^2 x^2} \text{ EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right] - \frac{1}{\sqrt{\frac{e (1+c x)}{-c d+e}}} c x (1+c x)\right.$$

$$\sqrt{\frac{e - c e x}{c d + e}} \sqrt{\frac{c d + c e x}{c d - e}} \left((c d + e) \text{ EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c d + c e x}{c d - e}}\right], \frac{c d - e}{c d + e}\right] - e \text{ EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c d + c e x}{c d - e}}\right], \frac{c d - e}{c d + e}\right]\right) +$$

$$\left.\left.\left.c e x \sqrt{\frac{c d + c e x}{c d + e}} \sqrt{1-c^2 x^2} \text{ EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]\right)\right) \Big/ \left(\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} \sqrt{c x} \left(-2+c^2 x^2\right)\right)\right)\right)$$

Problem 64: Unable to integrate problem.

$$\int \frac{x^2 \left(a + b \,\text{ArcCsc}[c\, x]\right)}{(d + e\, x)^{3/2}} \, dx$$

Optimal (type 4, 369 leaves, 16 steps):

$$-\frac{2 d^2 \left(a + b\,\text{ArcCsc}[c\,x]\right)}{e^3 \sqrt{d + e\,x}} - \frac{4 d \sqrt{d + e\,x} \left(a + b\,\text{ArcCsc}[c\,x]\right)}{e^3} +$$

$$\frac{2 (d + e\,x)^{3/2} \left(a + b\,\text{ArcCsc}[c\,x]\right)}{3 e^3} - \frac{4 b \sqrt{d + e\,x} \sqrt{1 - c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1 - c\,x}}{\sqrt{2}}\right], \frac{2 e}{c\,d + e}\right]}{3 c^2 e^2 \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d + e\,x)}{c\,d + e}}} +$$

$$\frac{20 b d \sqrt{\frac{c (d + e\,x)}{c\,d + e}} \sqrt{1 - c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1 - c\,x}}{\sqrt{2}}\right], \frac{2 e}{c\,d + e}\right]}{3 c^2 e^2 \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{d + e\,x}} + \frac{32 b d^2 \sqrt{\frac{c (d + e\,x)}{c\,d + e}} \sqrt{1 - c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1 - c\,x}}{\sqrt{2}}\right], \frac{2 e}{c\,d + e}\right]}{3 c\, e^3 \sqrt{1 - \frac{1}{c^2 x^2}}\, x \sqrt{d + e\,x}}$$

Result (type 9, 750 leaves):

$$-\frac{a\, d^3 \left(1+\frac{e x}{d}\right)^{3/2} \text{Beta}\left[-\frac{e x}{d}, 3, -\frac{1}{2}\right]}{e^3 (d+e x)^{3/2}}+\frac{1}{c^3} b \left(-\frac{c^2 \left(e+\frac{d}{x}\right)^2 x^2 \left(-\frac{4 \sqrt{1-\frac{1}{c^2 x^2}}}{3 e^2}+\frac{16 c\, d\, \text{ArcCsc}[c x]}{3 e^3}-\frac{2 c\, d\, \text{ArcCsc}[c x]}{e^2 \left(e+\frac{d}{x}\right)}-\frac{2 c\, x\, \text{ArcCsc}[c x]}{3 e^2}\right)}{(d+e x)^{3/2}}+\right.$$

$$\frac{1}{3 e^3 (d+e x)^{3/2}} 2 \left(e+\frac{d}{x}\right)^{3/2} (c x)^{3/2} \left(\frac{10 c\, d\, e \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c x)^{3/2}}+\right.$$

$$\frac{2 \left(8 c^2 d^2+e^2\right) \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c x)^{3/2}}-\Bigg(2 e \cos[2\, \text{ArcCsc}[c x]]$$

$$\left((c d+c e x) \left(-1+c^2 x^2\right)+c^2 d\, x \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]-\frac{1}{\sqrt{\frac{e (1+c x)}{-c d+e}}} c x (1+c x)\right.$$

$$\sqrt{\frac{e-c e x}{c d+e}} \sqrt{\frac{c d+c e x}{c d-e}} \left((c d+e)\, \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c d+c e x}{c d-e}}\right], \frac{c d-e}{c d+e}\right]-e\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c d+c e x}{c d-e}}\right], \frac{c d-e}{c d+e}\right]\right)+$$

$$\left.\left.\left.c e x \sqrt{\frac{c d+c e x}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]\right)\right) \Bigg/ \left(\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} \sqrt{c x} \left(-2+c^2 x^2\right)\right)\right)\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\, (a+b\, \text{ArcCsc}[c x])}{(d+e x)^{3/2}}\, dx$$

Optimal (type 4, 238 leaves, 11 steps):

$$\frac{2 d \left(a+b \operatorname{ArcCsc}[c x]\right)}{e^2 \sqrt{d+e x}}+\frac{2 \sqrt{d+e x} \left(a+b \operatorname{ArcCsc}[c x]\right)}{e^2}-\frac{4 b \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{c^2 e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}-\frac{8 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{c e^2 \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 226 leaves):

$$\frac{1}{e^2} 2 \left(\frac{a (2 d+e x)}{\sqrt{d+e x}}+\frac{b (2 d+e x) \operatorname{ArcCsc}[c x]}{\sqrt{d+e x}}-\frac{1}{c \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}} x} 2 i b \sqrt{\frac{e (1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right]-2 \operatorname{EllipticPi}\left[1+\frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right]\right)\right)$$

Problem 69: Unable to integrate problem.

$$\int \frac{x^3 \left(a+b \operatorname{ArcCsc}[c x]\right)}{(d+e x)^{5/2}} \, dx$$

Optimal (type 4, 602 leaves, 31 steps):

$$-\frac{4\,b\,d^2\left(1-c^2x^2\right)}{3\,c\,e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\;x\sqrt{d+e\,x}}+\frac{2\,d^3\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^4\left(d+e\,x\right)^{3/2}}-$$

$$\frac{6\,d^2\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4\sqrt{d+e\,x}}-\frac{6\,d\sqrt{d+e\,x}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^4}+\frac{2\left(d+e\,x\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^4}+$$

$$\frac{8\,b\,d^2\sqrt{d+e\,x}\sqrt{1-c^2x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\;x\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}-\frac{4\,b\left(2\,c^2d^2-e^2\right)\sqrt{d+e\,x}\sqrt{1-c^2x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2e^3\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\;x\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+$$

$$\frac{32\,b\,d\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\sqrt{1-c^2x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c^2e^3\sqrt{1-\frac{1}{c^2x^2}}\;x\sqrt{d+e\,x}}+\frac{64\,b\,d^2\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\sqrt{1-c^2x^2}\;\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e^4\sqrt{1-\frac{1}{c^2x^2}}\;x\sqrt{d+e\,x}}$$

Result (type 9, 887 leaves) :

$$\frac{a\, d^4 \left(1+\frac{e\, x}{d}\right)^{5/2} \text{Beta}\left[-\frac{e\, x}{d}, 4, -\frac{3}{2}\right]}{e^4 \left(d+e\, x\right)^{5/2}} +$$

$$\frac{1}{c^4} b \left(-\frac{1}{\left(d+e\, x\right)^{5/2}} c^3 \left(e+\frac{d}{x}\right)^3 x^3 \left(-\frac{4 \sqrt{1-\frac{1}{c^2 x^2}}}{3 e \left(-c^2 d^2+e^2\right)} + \frac{32\, c\, d\, \text{ArcCsc}[c\, x]}{3 e^4} - \frac{2\, c\, d\, \text{ArcCsc}[c\, x]}{3 e^2 \left(e+\frac{d}{x}\right)^2} - \frac{2\, c\, x\, \text{ArcCsc}[c\, x]}{3 e^3} - \right.\right.$$

$$\left.\frac{2 \left(-2 c^2 d^2 e \sqrt{1-\frac{1}{c^2 x^2}} - 7 c^3 d^3 \text{ArcCsc}[c\, x] + 7\, c\, d\, e^2 \text{ArcCsc}[c\, x]\right)}{3 e^3 \left(-c^2 d^2+e^2\right) \left(e+\frac{d}{x}\right)}\right) +$$

$$\frac{1}{3 \left(c\, d-e\right) e^4 \left(c\, d+e\right) \left(d+e\, x\right)^{5/2}} 2 \left(e+\frac{d}{x}\right)^{5/2} (c\, x)^{5/2} \left(\frac{2 \left(8 c^3 d^3 e - 8\, c\, d\, e^3\right) \sqrt{\frac{c\, d+c\, e\, x}{c\, d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\, x}}{\sqrt{2}}\right], \frac{2 e}{c\, d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c\, x)^{3/2}} + \right.$$

$$\frac{2 \left(16 c^4 d^4 - 16 c^2 d^2 e^2 - e^4\right) \sqrt{\frac{c\, d+c\, e\, x}{c\, d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c\, x}}{\sqrt{2}}\right], \frac{2 e}{c\, d+e}\right]}{\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} (c\, x)^{3/2}} + \left(2 e^3 \text{Cos}[2\, \text{ArcCsc}[c\, x]]\right.$$

$$\left((c\, d+c\, e\, x) \left(-1+c^2 x^2\right) + c^2 d\, x \sqrt{\frac{c\, d+c\, e\, x}{c\, d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\, x}}{\sqrt{2}}\right], \frac{2 e}{c\, d+e}\right] - \frac{1}{\sqrt{\frac{e (1+c\, x)}{-c\, d+e}}} c\, x\, (1+c\, x)\right.$$

$$\sqrt{\frac{e-c\, e\, x}{c\, d+e}} \sqrt{\frac{c\, d+c\, e\, x}{c\, d-e}} \left((c\, d+e)\, \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\, d+c\, e\, x}{c\, d-e}}\right], \frac{c\, d-e}{c\, d+e}\right] - e\, \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\, d+c\, e\, x}{c\, d-e}}\right], \frac{c\, d-e}{c\, d+e}\right]\right) +$$

$$\left.\left.\left.\left. c\, e\, x \sqrt{\frac{c\, d+c\, e\, x}{c\, d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c\, x}}{\sqrt{2}}\right], \frac{2 e}{c\, d+e}\right]\right)\right) \middle/ \left(\sqrt{1-\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} \sqrt{c\, x} \left(-2+c^2 x^2\right)\right)\right)\right)$$

Problem 70: Unable to integrate problem.

$$\int \frac{x^2 \left(a+b \,\text{ArcCsc}[c\, x]\right)}{(d+e\, x)^{5/2}} \, dx$$

Optimal (type 4, 440 leaves, 25 steps):

$$\frac{4 b d \left(1-c^2 x^2\right)}{3 c e \left(c^2 d^2-e^2\right) \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-\frac{2 d^2 \left(a+b \,\text{ArcCsc}[c x]\right)}{3 e^3 (d+e x)^{3/2}}+\frac{4 d \left(a+b \,\text{ArcCsc}[c x]\right)}{e^3 \sqrt{d+e x}}+$$

$$\frac{2 \sqrt{d+e x} \left(a+b \,\text{ArcCsc}[c x]\right)}{e^3}-\frac{4 b d \sqrt{d+e x} \sqrt{1-c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 e^2 \left(c^2 d^2-e^2\right) \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c (d+e x)}{c d+e}}}-$$

$$\frac{4 b \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{c^2 e^2 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-\frac{32 b d \sqrt{\frac{c (d+e x)}{c d+e}} \sqrt{1-c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c e^3 \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}$$

Result (type 9, 856 leaves):

$$-\frac{a\,d^3\left(1+\frac{e\,x}{d}\right)^{5/2}\text{Beta}\left[-\frac{e\,x}{d},3,-\frac{3}{2}\right]}{e^3\,(d+e\,x)^{5/2}}+\frac{1}{c^3}\,b\left(-\frac{1}{(d+e\,x)^{5/2}}\right.$$

$$c^3\left(e+\frac{d}{x}\right)^3 x^3\left(\frac{4\,c\,d\sqrt{1-\frac{1}{c^2x^2}}}{3\,e^2\left(-c^2d^2+e^2\right)}-\frac{16\,\text{ArcCsc}[c\,x]}{3\,e^3}+\frac{2\,\text{ArcCsc}[c\,x]}{3\,e\left(e+\frac{d}{x}\right)^2}+\frac{4\left(-c\,d\,e\sqrt{1-\frac{1}{c^2x^2}}-2\,c^2d^2\,\text{ArcCsc}[c\,x]+2\,e^2\,\text{ArcCsc}[c\,x]\right)}{3\,e^2\left(-c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)-$$

$$\frac{1}{3\,(c\,d-e)\,e^3\,(c\,d+e)\,(d+e\,x)^{5/2}}\,2\left(e+\frac{d}{x}\right)^{5/2}(c\,x)^{5/2}\left(\frac{2\left(3\,c^2d^2e-3\,e^3\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\right.$$

$$\frac{2\left(8\,c^3d^3-9\,c\,d\,e^2\right)\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}+\Big(2\,c\,d\,e\,\text{Cos}[2\,\text{ArcCsc}[c\,x]]$$

$$\left((c\,d+c\,e\,x)\left(-1+c^2x^2\right)+c^2d\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]-\frac{1}{\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}}c\,x\,(1+c\,x)\right.$$

$$\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\left((c\,d+e)\,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]-e\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]\right)+$$

$$\left.\left.c\,e\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\right)\Big/\left(\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\sqrt{c\,x}\left(-2+c^2x^2\right)\right)\Big)\Big)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 314 leaves, 19 steps):

$$-\frac{4\,b\,(1-c^2 x^2)}{3\,c\,(c^2 d^2-e^2)\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{d+e\,x}}+\frac{2\,d\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e^2\,(d+e\,x)^{3/2}}-\frac{2\,(a+b\,\text{ArcCsc}[c\,x])}{e^2\,\sqrt{d+e\,x}}+$$

$$\frac{4\,b\,\sqrt{d+e\,x}\,\sqrt{1-c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,e\,(c^2 d^2-e^2)\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+\frac{8\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\sqrt{1-c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{3\,c\,e^2\,\sqrt{1-\frac{1}{c^2 x^2}}\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 345 leaves):

$$\frac{4\,b\,c\,\sqrt{1-\frac{1}{c^2 x^2}}\,x}{3\,(c^2 d^2-e^2)\sqrt{d+e\,x}}-\frac{2\,a\,(2\,d+3\,e\,x)}{3\,e^2\,(d+e\,x)^{3/2}}-\frac{2\,b\,(2\,d+3\,e\,x)\,\text{ArcCsc}[c\,x]}{3\,e^2\,(d+e\,x)^{3/2}}+\frac{1}{3\,c^2\,d\,e^2\sqrt{1-\frac{1}{c^2 x^2}}\,x}$$

$$4\,i\,b\,\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}\,\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\left(c\,d\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]-\right.$$

$$\left.c\,d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]+2\,(c\,d+e)\,\text{EllipticPi}\left[1+\frac{e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d+e}}\,\sqrt{d+e\,x}\right],\frac{c\,d+e}{c\,d-e}\right]\right)$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{(d+e\,x)^{5/2}}\,dx$$

Optimal (type 4, 298 leaves, 12 steps):

$$\frac{4\,b\,e\,\left(1-c^2\,x^2\right)}{3\,c\,d\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\,x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e\,\left(d+e\,x\right)^{3/2}}-$$

$$\frac{4\,b\,\sqrt{d+e\,x}\,\,\sqrt{1-c^2\,x^2}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{c\,d+e}\right]}{3\,d\,\left(c^2\,d^2-e^2\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\,x\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}}+\frac{4\,b\,\sqrt{\frac{c\,(d+e\,x)}{c\,d+e}}\,\,\sqrt{1-c^2\,x^2}\,\,\text{EllipticPi}\left[2,\,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{c\,d+e}\right]}{3\,c\,d\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\,x\,\sqrt{d+e\,x}}$$

Result (type 4, 725 leaves) :

$$-\frac{2\,a}{3\,e\,(d+e\,x)^{3/2}}+\frac{1}{c}\,b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3 x^3\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3\,c\,d\,(c^2d^2-e^2)}+\frac{2\,\text{ArcCsc}[c\,x]}{3\,c^2d^2\,e}+\frac{2\,e\,\text{ArcCsc}[c\,x]}{3\,c^2d^2\left(e+\frac{d}{x}\right)^2}-\frac{4\left(-c\,d\,e\sqrt{1-\frac{1}{c^2x^2}}+c^2d^2\,\text{ArcCsc}[c\,x]-e^2\,\text{ArcCsc}[c\,x]\right)}{3\,c^2d^2\,(c^2d^2-e^2)\left(e+\frac{d}{x}\right)}\right)}{(d+e\,x)^{5/2}}+\frac{1}{3\,(c\,d-e)\,e\,(c\,d+e)\,(d+e\,x)^{5/2}}\right.$$

$$2\left(e+\frac{d}{x}\right)^{5/2}(c\,x)^{5/2}\left(\frac{2\,c\,d\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\,(c\,x)^{3/2}}-\left(2\,e\,\text{Cos}[2\,\text{ArcCsc}[c\,x]]\right.\right.$$

$$\left((c\,d+c\,e\,x)\,(-1+c^2x^2)+c^2d\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]-\frac{1}{\sqrt{\frac{e\,(1+c\,x)}{-c\,d+e}}}c\,x\,(1+c\,x)\right.$$

$$\sqrt{\frac{e-c\,e\,x}{c\,d+e}}\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\left((c\,d+e)\,\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]-e\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{c\,d+c\,e\,x}{c\,d-e}}\right],\frac{c\,d-e}{c\,d+e}\right]\right)+$$

$$\left.\left.\left.c\,e\,x\sqrt{\frac{c\,d+c\,e\,x}{c\,d+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-c\,x}}{\sqrt{2}}\right],\frac{2\,e}{c\,d+e}\right]\right)\right)\Big/\left(c\,d\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\sqrt{c\,x}\,(-2+c^2x^2)\right)\right)\right)$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2\,(a+b\,\text{ArcCsc}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 565 leaves, 25 steps):

$$\frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{e}+\frac{b\,\text{ArcTanh}\left[\sqrt{1-\frac{1}{c^2 x^2}}\right]}{c\,e}-\frac{\sqrt{-d}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{\sqrt{-d}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-\frac{\sqrt{-d}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{\sqrt{-d}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-\frac{i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+$$

$$\frac{i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}-\frac{i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}+\frac{i\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^{3/2}}$$

Result (type 4, 1260 leaves) :

$$\frac{1}{4\,c\,e^{3/2}}\,i\,\Bigg(-4\,i\,a\,c\,\sqrt{e}\,x-4\,i\,b\,c\,\sqrt{e}\,x\,\text{ArcCsc}[c\,x]+$$

$$4\,i\,a\,c\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+8\,i\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]-$$

$$8\,i\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]+b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,c\,\sqrt{d}\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+2\,b\,c\,\sqrt{d}\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2 b c \sqrt{d}\, \text{ArcCsc}[c x]\, \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+4 b c \sqrt{d}\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+$$

$$b c \sqrt{d}\, \pi\, \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-2 b c \sqrt{d}\, \text{ArcCsc}[c x]\, \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$4 b c \sqrt{d}\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+b c \sqrt{d}\, \pi\, \text{Log}\left[\sqrt{e}-\frac{i \sqrt{d}}{x}\right]-$$

$$b c \sqrt{d}\, \pi\, \text{Log}\left[\sqrt{e}+\frac{i \sqrt{d}}{x}\right]-4 i b \sqrt{e}\, \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right]+4 i b \sqrt{e}\, \text{Log}\left[\text{Sin}\left[\frac{1}{2} \text{ArcCsc}[c x]\right]\right]+$$

$$2 i b c \sqrt{d}\, \text{PolyLog}\left[2, \frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-2 i b c \sqrt{d}\, \text{PolyLog}\left[2, \frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]-$$

$$\left.2 i b c \sqrt{d}\, \text{PolyLog}\left[2, -\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]+2 i b c \sqrt{d}\, \text{PolyLog}\left[2, \frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right]\right)$$

Problem 99: Result more than twice size of optimal antiderivative.

$$\int \frac{x\, (a+b\, \text{ArcCsc}[c x])}{d+e x^2}\, dx$$

Optimal (type 4, 507 leaves, 26 steps):

$$\frac{(a+b\, \text{ArcCsc}[c x])\, \text{Log}\left[1-\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 e}+\frac{(a+b\, \text{ArcCsc}[c x])\, \text{Log}\left[1+\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 e}+\frac{(a+b\, \text{ArcCsc}[c x])\, \text{Log}\left[1-\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 e}+$$

$$\frac{(a+b\, \text{ArcCsc}[c x])\, \text{Log}\left[1+\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 e}-\frac{(a+b\, \text{ArcCsc}[c x])\, \text{Log}\left[1-e^{2 i \text{ArcCsc}[c x]}\right]}{e}-\frac{i b\, \text{PolyLog}\left[2,-\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 e}-$$

$$\frac{i b\, \text{PolyLog}\left[2,\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 e}-\frac{i b\, \text{PolyLog}\left[2,-\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 e}-\frac{i b\, \text{PolyLog}\left[2,\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 e}+\frac{i b\, \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]}{2 e}$$

Result (type 4, 1123 leaves):

$$\frac{1}{8 e}\Bigg(i b \pi^2 - 4 i b \pi \text{ArcCsc}[c x] + 8 i b \text{ArcCsc}[c x]^2 - 16 i b \text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{ArcTan}\Big[\frac{\left(-i c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\Big] -$$

$$16 i b \text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{ArcTan}\Big[\frac{\left(i c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\text{ArcCsc}[c x]\right)\right]}{\sqrt{c^2 d+e}}\Big] - 2 b \pi \text{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] +$$

$$4 b \text{ArcCsc}[c x] \text{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] - 8 b \text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] -$$

$$2 b \pi \text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] + 4 b \text{ArcCsc}[c x] \text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] -$$

$$8 b \text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] - 2 b \pi \text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] +$$

$$4 b \text{ArcCsc}[c x] \text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] + 8 b \text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] -$$

$$2 b \pi \text{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] + 4 b \text{ArcCsc}[c x] \text{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] +$$

$$8 b \text{ArcSin}\Big[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big] \text{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] - 8 b \text{ArcCsc}[c x] \text{Log}\Big[1-e^{2 i \text{ArcCsc}[c x]}\Big] +$$

$$2 b \pi \text{Log}\Big[\sqrt{e}-\frac{i\sqrt{d}}{x}\Big] + 2 b \pi \text{Log}\Big[\sqrt{e}+\frac{i\sqrt{d}}{x}\Big] + 4 a \text{Log}\left[d+e x^2\right] + 4 i b \text{PolyLog}\Big[2, \frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] +$$

$$4 i b \text{PolyLog}\Big[2, \frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] + 4 i b \text{PolyLog}\Big[2, -\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i \text{ArcCsc}[c x]}}{c\sqrt{d}}\Big] +$$

$$4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\Bigg)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 529 leaves, 19 steps):

$$-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}$$

Result (type 4, 1068 leaves):

$$-\frac{1}{4\sqrt{d}\sqrt{e}}\,\mathrm{i}\,\Bigg(4\,\mathrm{i}\,a\,\mathrm{ArcTan}\Big[\frac{\sqrt{e}\,x}{\sqrt{d}}\Big]+8\,\mathrm{i}\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{ArcTan}\Big[\frac{\left(-\mathrm{i}\,c\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\Big[\frac{1}{4}\left(\pi+2\,\mathrm{ArcCsc}[c\,x]\right)\Big]}{\sqrt{c^2 d+e}}\Big]-$$

$$8\,\mathrm{i}\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{ArcTan}\Big[\frac{\left(\mathrm{i}\,c\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\Big[\frac{1}{4}\left(\pi+2\,\mathrm{ArcCsc}[c\,x]\right)\Big]}{\sqrt{c^2 d+e}}\Big]+b\,\pi\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$2\,b\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+4\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$b\,\pi\,\mathrm{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+2\,b\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$4\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-b\,\pi\,\mathrm{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,b\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+4\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$b\,\pi\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-2\,b\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$4\,b\,\mathrm{ArcSin}\Big[\frac{\sqrt{1-\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\mathrm{Log}\Big[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+b\,\pi\,\mathrm{Log}\Big[\sqrt{e}-\frac{\mathrm{i}\sqrt{d}}{x}\Big]-b\,\pi\,\mathrm{Log}\Big[\sqrt{e}+\frac{\mathrm{i}\sqrt{d}}{x}\Big]+$$

$$2\,\mathrm{i}\,b\,\mathrm{PolyLog}\Big[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-2\,\mathrm{i}\,b\,\mathrm{PolyLog}\Big[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$2\,\mathrm{i}\,b\,\mathrm{PolyLog}\Big[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]+2\,\mathrm{i}\,b\,\mathrm{PolyLog}\Big[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\Big]\Bigg)$$

Problem 101: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\text{ArcCsc}[c\,x]}{x\,(d + e\,x^2)}\,dx$$

Optimal (type 4, 479 leaves, 19 steps):

$$\frac{i\,(a + b\,\text{ArcCsc}[c\,x])^2}{2\,b\,d} - \frac{(a + b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1 - \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} - \sqrt{c^2\,d + e}}\right]}{2\,d} - \frac{(a + b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1 + \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} - \sqrt{c^2\,d + e}}\right]}{2\,d} -$$
$$\frac{(a + b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1 - \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} + \sqrt{c^2\,d + e}}\right]}{2\,d} - \frac{(a + b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1 + \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} + \sqrt{c^2\,d + e}}\right]}{2\,d} + \frac{i\,b\,\text{PolyLog}\left[2, -\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} - \sqrt{c^2\,d + e}}\right]}{2\,d} +$$
$$\frac{i\,b\,\text{PolyLog}\left[2, \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} - \sqrt{c^2\,d + e}}\right]}{2\,d} + \frac{i\,b\,\text{PolyLog}\left[2, -\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} + \sqrt{c^2\,d + e}}\right]}{2\,d} + \frac{i\,b\,\text{PolyLog}\left[2, \frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e} + \sqrt{c^2\,d + e}}\right]}{2\,d}$$

Result (type 4, 1089 leaves):

$$-\frac{1}{8\,d}\left(i\,b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+4\,i\,b\,\text{ArcCsc}[c\,x]^2-16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]-\right.$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]-2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-2\,b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]-8\,a\,\text{Log}[x]+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.4\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{x^2\,(d+e\,x^2)}\,dx$$

Optimal (type 4, 572 leaves, 24 steps):

$$-\frac{b\,c\,\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{d\,x}-\frac{b\,\text{ArcCsc}[c\,x]}{d\,x}-\frac{\sqrt{e}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{\sqrt{e}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\,\text{ArcCsc}[c\,x])\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}-\frac{i\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}+$$
$$\frac{i\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}-\frac{i\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}+\frac{i\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,(-d)^{3/2}}$$

Result (type 4, 1241 leaves):

$$-\frac{a}{d\,x}-\frac{a\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{d^{3/2}}+b\left(-\frac{c\,\sqrt{1-\frac{1}{c^2x^2}}\,x+\text{ArcCsc}[c\,x]}{d\,x}+\right.$$
$$\frac{1}{16\,d^{3/2}}\sqrt{e}\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\,(\pi+2\,\text{ArcCsc}[c\,x])\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$
$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$
$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1-e^{2\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}\right]-4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[\sqrt{e}+\frac{\mathrm{i}\sqrt{d}}{x}\right]+8\,\mathrm{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\mathrm{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\mathrm{PolyLog}\left[2,e^{2\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}\right]\right)-$$

$$\frac{1}{16\,d^{3/2}}\sqrt{e}\left(\pi^2-4\,\pi\,\mathrm{ArcCsc}[c\,x]+8\,\mathrm{ArcCsc}[c\,x]^2-32\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{ArcTan}\left[\frac{\left(\mathrm{i}\,c\sqrt{d}+\sqrt{e}\right)\mathrm{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathrm{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,\mathrm{i}\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\mathrm{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]\,\mathrm{Log}\left[1-e^{2\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}\right]-4\,\mathrm{i}\,\pi\,\mathrm{Log}\left[\sqrt{e}-\frac{\mathrm{i}\sqrt{d}}{x}\right]+8\,\mathrm{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.\left.8\,\mathrm{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\mathrm{PolyLog}\left[2,e^{2\,\mathrm{i}\,\mathrm{ArcCsc}[c\,x]}\right]\right)\right)$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5 \left(a + b \text{ArcCsc}[c x]\right)}{\left(d + e x^2\right)^2} \, dx$$

Optimal (type 4, 628 leaves, 31 steps):

$$\frac{b \sqrt{1 - \frac{1}{c^2 x^2}}\, x}{2 c e^2} + \frac{d \left(a + b \text{ArcCsc}[c x]\right)}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 \left(a + b \text{ArcCsc}[c x]\right)}{2 e^2} - \frac{b d \text{ArcTan}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}\, x}\right]}{2 e^{5/2} \sqrt{c^2 d + e}} - \frac{d \left(a + b \text{ArcCsc}[c x]\right) \text{Log}\left[1 - \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} -$$

$$\frac{d \left(a + b \text{ArcCsc}[c x]\right) \text{Log}\left[1 + \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d \left(a + b \text{ArcCsc}[c x]\right) \text{Log}\left[1 - \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d \left(a + b \text{ArcCsc}[c x]\right) \text{Log}\left[1 + \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} +$$

$$\frac{2 d \left(a + b \text{ArcCsc}[c x]\right) \text{Log}\left[1 - e^{2 i \text{ArcCsc}[c x]}\right]}{e^3} + \frac{i b d \text{PolyLog}\left[2, -\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} + \frac{i b d \text{PolyLog}\left[2, \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} +$$

$$\frac{i b d \text{PolyLog}\left[2, -\frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} + \frac{i b d \text{PolyLog}\left[2, \frac{i c \sqrt{-d}\, e^{i \text{ArcCsc}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{i b d \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]}{e^3}$$

Result (type 4, 1604 leaves):

$$\frac{a x^2}{2 e^2} - \frac{a d^2}{2 e^3 \left(d + e x^2\right)} - \frac{a d \text{Log}\left[d + e x^2\right]}{e^3} + b \left(\frac{x \left(\sqrt{1 - \frac{1}{c^2 x^2}} + c x \text{ArcCsc}[c x]\right)}{2 c e^2} - \right.$$

$$\frac{i\, d^{3/2} \left(-\frac{\text{ArcCsc}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)}{4\,e^{5/2}}+\frac{i\, d^{3/2} \left(-\frac{\text{ArcCsc}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2 d-e}\,\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)}{4\,e^{5/2}}-$$

$$\frac{1}{8\,e^3}\, i\, d \left(\pi^2-4\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)-$$

$$\frac{1}{8 e^3} i d \left(\pi^2 - 4 \pi \text{ArcCsc}[c x] + 8 \text{ArcCsc}[c x]^2 - 32 \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(i c \sqrt{d} + \sqrt{e}\right) \text{Cot}\left[\frac{1}{4} (\pi + 2 \text{ArcCsc}[c x])\right]}{\sqrt{c^2 d + e}}\right] + \right.$$

$$4 i \pi \text{Log}\left[1 + \frac{\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] - 8 i \text{ArcCsc}[c x] \text{Log}\left[1 + \frac{\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] +$$

$$16 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] + 4 i \pi \text{Log}\left[1 - \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] -$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1 - \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] - 16 i \text{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] +$$

$$8 i \text{ArcCsc}[c x] \text{Log}\left[1 - e^{2 i \text{ArcCsc}[c x]}\right] - 4 i \pi \text{Log}\left[\sqrt{e} - \frac{i \sqrt{d}}{x}\right] + 8 \text{PolyLog}\left[2, \frac{\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] +$$

$$\left. 8 \text{PolyLog}\left[2, \frac{\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-i \text{ArcCsc}[c x]}}{c \sqrt{d}}\right] + 4 \text{PolyLog}\left[2, e^{2 i \text{ArcCsc}[c x]}\right]\right)$$

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 590 leaves, 29 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e\left(e+\frac{d}{x^2}\right)}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2 x^2}}\,x}\right]}{2\,e^{3/2}\sqrt{c^2 d+e}}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^2}+$$
$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^2}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^2}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^2}-$$
$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^2}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^2}-\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,e^2}-$$
$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^2}-\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,e^2}+\frac{i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{2\,e^2}$$

Result (type 4, 1442 leaves):

$$\frac{1}{8\,e^2}\left(i\,b\,\pi^2+\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+\frac{2\,b\,\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}-i\,\sqrt{e}\,x}+\frac{2\,b\,\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}+i\,\sqrt{e}\,x}+\right.$$
$$8\,i\,b\,\text{ArcCsc}[c\,x]^2-4\,b\,\text{ArcSin}\left[\frac{1}{c\,x}\right]-16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]-$$
$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]-2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-2\,b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-8\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]+2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]+\frac{2\,b\,\sqrt{e}\,\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\,\left(-i\,c\,\sqrt{d}-\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+\frac{2\,b\,\sqrt{e}\,\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(-\sqrt{e}+c\,\left(-i\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d-e}\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,i\,b\,\text{PolyLog}\left[2,\,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,\,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.4\,i\,b\,\text{PolyLog}\left[2,\,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,\,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,b\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)$$

Problem 105: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x^2)^2}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e\,(d+e\,x^2)}-\frac{b\,c\,x\,\text{ArcTan}\left[\sqrt{-1+c^2\,x^2}\right]}{2\,d\,e\,\sqrt{c^2\,x^2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2\,x^2}}{\sqrt{c^2\,d+e}}\right]}{2\,d\,\sqrt{e}\,\sqrt{c^2\,d+e}\,\sqrt{c^2\,x^2}}$$

Result (type 3, 286 leaves):

$$-\frac{1}{4\,e}\left(\frac{2\,a}{d+e\,x^2}+\frac{2\,b\,\text{ArcCsc}[c\,x]}{d+e\,x^2}-\frac{2\,b\,\text{ArcSin}\left[\frac{1}{c\,x}\right]}{d}+\frac{b\,\sqrt{e}\,\text{Log}\left[\frac{4\,i\,d\,e-4\,c\,d\,\sqrt{e}\,\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x}{b\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{d\,\sqrt{-c^2\,d-e}}+\frac{b\,\sqrt{e}\,\text{Log}\left[\frac{4\,i\,\left(-d\,e+c\,d\,\sqrt{e}\,\left(i\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)}{b\,\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\sqrt{-c^2\,d-e}}\right)$$

Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{x\,(d+e\,x^2)^2}\,dx$$

Optimal (type 4, 566 leaves, 24 steps):

$$-\frac{e\left(a+b\,\text{ArcCsc}[c\,x]\right)}{2\,d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\,\text{ArcCsc}[c\,x]\right)^2}{2\,b\,d^2}+\frac{b\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]}{2\,d^2\sqrt{c^2 d+e}}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,d^2}-$$

$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,d^2}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,d^2}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,d^2}+$$

$$\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,d^2}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2\,d^2}+\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,d^2}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2\,d^2}$$

Result (type 4, 1408 leaves):

$$\frac{1}{8\,d^2}\left(-i\,b\,\pi^2+\frac{4\,a\,d}{d+e\,x^2}+4\,i\,b\,\pi\,\text{ArcCsc}[c\,x]+\frac{2\,b\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}-i\sqrt{e}\,x}+\frac{2\,b\sqrt{d}\,\text{ArcCsc}[c\,x]}{\sqrt{d}+i\sqrt{e}\,x}-\right.$$

$$4\,i\,b\,\text{ArcCsc}[c\,x]^2-4\,b\,\text{ArcSin}\left[\frac{1}{c\,x}\right]+16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+$$

$$16\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+2\,b\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,b\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-4\,b\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-2\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-2\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$8\,a\,\text{Log}[x]+\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}+\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}-$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]-4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$\left.4\,i\,b\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,b\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 107: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 803 leaves, 51 steps) :

$$-\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{x\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}+\frac{b\,\text{ArcTanh}\left[\sqrt{1-\frac{1}{c^2\,x^2}}\right]}{c\,e^2}+$$

$$\frac{b\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}}\right]}{4\,e^2\sqrt{c^2\,d+e}}+\frac{b\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}}\right]}{4\,e^2\sqrt{c^2\,d+e}}-\frac{3\sqrt{-d}\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\sqrt{-d}\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\sqrt{-d}\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\sqrt{-d}\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,i\,b\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,i\,b\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}-\frac{3\,i\,b\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,i\,b\sqrt{-d}\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{4\,e^{5/2}}$$

Result (type 4, 1634 leaves):

$$\frac{a\,x}{e^2}+\frac{a\,d\,x}{2\,e^2\left(d+e\,x^2\right)}-\frac{3\,a\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,e^{5/2}}+$$

$$b\left(-\frac{d\left(-\frac{\text{ArcCsc}[c\,x]}{-i\sqrt{d}\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(-i\,c\sqrt{d}-\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)}{4\,e^2}-\frac{d\left(-\frac{\text{ArcCsc}[c\,x]}{i\sqrt{d}\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e}+c\left(-i\,c\sqrt{d}+\sqrt{-c^2 d-e}\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d-e}\left(\sqrt{d}-i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d-e}}\right)}{\sqrt{d}}\right)}{4\,e^2}+\right.$$

$$\frac{1}{32\,e^{5/2}}3\sqrt{d}\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\Big[2,\,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+4\,\text{PolyLog}\Big[2,\,e^{2\,i\,\text{ArcCsc}[c\,x]}\Big]\Bigg)-$$

$$\frac{1}{32\,e^{5/2}}\,3\,\sqrt{d}\,\Bigg(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\Big[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\Big]}{\sqrt{c^2\,d+e}}\Big]+$$

$$4\,i\,\pi\,\text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+4\,i\,\pi\,\text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]-16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\Big[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\Big]-4\,i\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\Big]+8\,\text{PolyLog}\Big[2,\,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$8\,\text{PolyLog}\Big[2,\,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\Big]+4\,\text{PolyLog}\Big[2,\,e^{2\,i\,\text{ArcCsc}[c\,x]}\Big]\Bigg)+\frac{1}{c\,e^2}$$

$$\left(\frac{1}{2}\,\text{ArcCsc}[c\,x]\,\text{Cot}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]+\text{Log}\left[\text{Cos}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right]-\text{Log}\left[\text{Sin}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right]+\frac{1}{2}\,\text{ArcCsc}[c\,x]\,\text{Tan}\left[\frac{1}{2}\,\text{ArcCsc}[c\,x]\right]\right)\Bigg)$$

Problem 111: Result more than twice size of optimal antiderivative.

$$\int\frac{x^5\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 727 leaves, 33 steps):

$$\frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2 x^2}}}{8\,e^2\,\left(c^2\,d+e\right)\,\left(e+\frac{d}{x^2}\right)\,x}-\frac{a+b\,\text{ArcCsc}[c\,x]}{4\,e\,\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\,\text{ArcCsc}[c\,x]}{2\,e^2\,\left(e+\frac{d}{x^2}\right)}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2\,d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}\right]}{2\,e^{5/2}\,\sqrt{c^2\,d+e}}+\frac{b\,\left(c^2\,d+2\,e\right)\,\text{ArcTan}\left[\frac{\sqrt{c^2\,d+e}}{c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}\right]}{8\,e^{5/2}\,\left(c^2\,d+e\right)^{3/2}}+$$

$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^3}+\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^3}+$$

$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1+\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^3}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{e^3}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^3}-$$

$$\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2\,d+e}}\right]}{2\,e^3}-\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^3}-\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\,\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2\,d+e}}\right]}{2\,e^3}+\frac{i\,b\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]}{2\,e^3}$$

Result (type 4, 2053 leaves) :

$$-\frac{a\,d^2}{4\,e^3\,\left(d+e\,x^2\right)^2}+\frac{a\,d}{e^3\,\left(d+e\,x^2\right)}+\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,e^3}+$$

$$b\left(\frac{7\,i\,\sqrt{d}\,\left(-\frac{\text{ArcCsc}[c\,x]}{-i\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\,\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\,\left(-i\,c\,\sqrt{d}-\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d-e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right)}{16\,e^{5/2}}-\frac{7\,i\,\sqrt{d}\,\left(-\frac{\text{ArcCsc}[c\,x]}{i\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\,\left(\frac{\text{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(-\sqrt{e}+c\,\left(-i\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d-e}\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right)}{16\,e^{5/2}}-\frac{1}{16\,e^{5/2}}\right.$$

$$d\left(\frac{i\, c\sqrt{e}\,\sqrt{1-\frac{1}{c^2 x^2}}\; x}{\sqrt{d}\,\left(c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\; x\right)}-\frac{\text{ArcCsc}[c\, x]}{\sqrt{e}\,\left(-i\sqrt{d}+\sqrt{e}\; x\right)^2}-\frac{\text{ArcSin}\left[\frac{1}{c\, x}\right]}{d\sqrt{e}}+\frac{i\left(2 c^2 d+e\right)\text{Log}\left[\frac{4 d\sqrt{e}\,\sqrt{c^2 d+e}\,\left(i\sqrt{e}+c\left(c\sqrt{d}-\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\left(2 c^2 d+e\right)\left(-i\sqrt{d}+\sqrt{e}\; x\right)}\right]}{d\left(c^2 d+e\right)^{3/2}}\right)-$$

$$\frac{1}{16\, e^{5/2}}$$

$$d\left(-\frac{i\, c\sqrt{e}\,\sqrt{1-\frac{1}{c^2 x^2}}\; x}{\sqrt{d}\,\left(c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e}\; x\right)}-\frac{\text{ArcCsc}[c\, x]}{\sqrt{e}\,\left(i\sqrt{d}+\sqrt{e}\; x\right)^2}-\frac{\text{ArcSin}\left[\frac{1}{c\, x}\right]}{d\sqrt{e}}+\frac{i\left(2 c^2 d+e\right)\text{Log}\left[-\frac{4 d\sqrt{e}\,\sqrt{c^2 d+e}\,\left(-i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}\right)x\right)}{\left(2 c^2 d+e\right)\left(i\sqrt{d}+\sqrt{e}\; x\right)}\right]}{d\left(c^2 d+e\right)^{3/2}}\right)+$$

$$\frac{1}{16\, e^3}\, i\left(\pi^2-4\pi\,\text{ArcCsc}[c\, x]+8\,\text{ArcCsc}[c\, x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\, c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\, x]\right)\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$4\, i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]-8\, i\,\text{ArcCsc}[c\, x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]+$$

$$16\, i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]+4\, i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]-$$

$$8\, i\,\text{ArcCsc}[c\, x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]-16\, i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]+$$

$$8\, i\,\text{ArcCsc}[c\, x]\,\text{Log}\left[1-e^{2\, i\,\text{ArcCsc}[c\, x]}\right]-4\, i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\, x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\Bigg)+$$

$$\frac{1}{16\,e^3}\,i\left(\pi^2-4\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2 d+e}}\right]+\right.$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2 d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\Bigg)\Bigg)$$

Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^3\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$-\frac{b\,c\,x\sqrt{-1+c^2 x^2}}{8\,e\left(c^2 d+e\right)\sqrt{c^2 x^2}\left(d+e\,x^2\right)}+\frac{x^4\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,d\left(d+e\,x^2\right)^2}+\frac{b\,c\left(c^2 d+2\,e\right)x\,\text{ArcTan}\left[\frac{\sqrt{e}\sqrt{-1+c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8\,d\,e^{3/2}\left(c^2 d+e\right)^{3/2}\sqrt{c^2 x^2}}$$

Result (type 3, 390 leaves):

$$\frac{1}{16 e^2}\left(\frac{4 a d}{\left(d+e x^2\right)^2}-\frac{8 a}{d+e x^2}-\frac{2 b c e \sqrt{1-\frac{1}{c^2 x^2}}\, x}{\left(c^2 d+e\right)\left(d+e x^2\right)}-\frac{4 b \left(d+2 e x^2\right) \text{ArcCsc}[c x]}{\left(d+e x^2\right)^2}+\frac{4 b \text{ArcSin}\left[\frac{1}{c x}\right]}{d}+\right.$$

$$\left.\frac{b \sqrt{e}\left(c^2 d+2 e\right) \text{Log}\left[\frac{16 d \sqrt{-c^2 d-e}\, e^{3/2}\left(i \sqrt{e}+c\left(c \sqrt{d}-i \sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{b\left(c^2 d+2 e\right)\left(\sqrt{d}+i \sqrt{e}\, x\right)}\right]}{d\left(-c^2 d-e\right)^{3/2}}+\frac{b \sqrt{e}\left(c^2 d+2 e\right) \text{Log}\left[-\frac{16 d \sqrt{-c^2 d-e}\, e^{3/2}\left(-\sqrt{e}+c\left(-i c \sqrt{d}+\sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{b\left(c^2 d+2 e\right)\left(i \sqrt{d}+\sqrt{e}\, x\right)}\right]}{d\left(-c^2 d-e\right)^{3/2}}\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\left(a+b \text{ArcCsc}[c x]\right)}{\left(d+e x^2\right)^3}\, dx$$

Optimal (type 3, 193 leaves, 8 steps):

$$\frac{b c x \sqrt{-1+c^2 x^2}}{8 d\left(c^2 d+e\right) \sqrt{c^2 x^2}\left(d+e x^2\right)}-\frac{a+b \text{ArcCsc}[c x]}{4 e\left(d+e x^2\right)^2}-\frac{b c x \text{ArcTan}\left[\sqrt{-1+c^2 x^2}\right]}{4 d^2 e \sqrt{c^2 x^2}}+\frac{b c\left(3 c^2 d+2 e\right) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8 d^2 \sqrt{e}\left(c^2 d+e\right)^{3/2} \sqrt{c^2 x^2}}$$

Result (type 3, 385 leaves):

$$\frac{1}{16}\left(-\frac{4 a}{e\left(d+e x^2\right)^2}+\frac{2 b c \sqrt{1-\frac{1}{c^2 x^2}}\, x}{d\left(c^2 d+e\right)\left(d+e x^2\right)}-\frac{4 b \text{ArcCsc}[c x]}{e\left(d+e x^2\right)^2}+\frac{4 b \text{ArcSin}\left[\frac{1}{c x}\right]}{d^2 e}+\right.$$

$$\left.\frac{b\left(3 c^2 d+2 e\right) \text{Log}\left[\frac{16 d^2 \sqrt{-c^2 d-e} \sqrt{e}\left(i \sqrt{e}+c\left(c \sqrt{d}-i \sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{b\left(3 c^2 d+2 e\right)\left(\sqrt{d}+i \sqrt{e}\, x\right)}\right]}{d^2\left(-c^2 d-e\right)^{3/2} \sqrt{e}}+\frac{b\left(3 c^2 d+2 e\right) \text{Log}\left[-\frac{16 d^2 \sqrt{-c^2 d-e} \sqrt{e}\left(-\sqrt{e}+c\left(-i c \sqrt{d}+\sqrt{-c^2 d-e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{b\left(3 c^2 d+2 e\right)\left(i \sqrt{d}+\sqrt{e}\, x\right)}\right]}{d^2\left(-c^2 d-e\right)^{3/2} \sqrt{e}}\right)$$

Problem 114: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{x\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 704 leaves, 28 steps):

$$-\frac{b\,c\,e\sqrt{1-\frac{1}{c^2x^2}}}{8\,d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\,\text{ArcCsc}[c\,x]\right)}{4\,d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\,\text{ArcCsc}[c\,x]\right)}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\,\text{ArcCsc}[c\,x]\right)^2}{2\,b\,d^3}+\frac{b\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]}{d^3\sqrt{c^2d+e}}-$$

$$\frac{b\sqrt{e}\left(c^2d+2\,e\right)\text{ArcTan}\left[\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right]}{8\,d^3\left(c^2d+e\right)^{3/2}}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,d^3}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,d^3}-$$

$$\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,d^3}-\frac{\left(a+b\,\text{ArcCsc}[c\,x]\right)\text{Log}\left[1+\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,d^3}+\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,d^3}+$$

$$\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2\,d^3}+\frac{i\,b\,\text{PolyLog}\left[2,-\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,d^3}+\frac{i\,b\,\text{PolyLog}\left[2,\frac{i\,c\sqrt{-d}\,e^{i\,\text{ArcCsc}[c\,x]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2\,d^3}$$

Result (type 4, 2114 leaves):

$$\frac{a}{4\,d\left(d+e\,x^2\right)^2}+\frac{a}{2\,d^2\left(d+e\,x^2\right)}+\frac{a\,\text{Log}[x]}{d^3}-\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,d^3}+$$

$$b \left(\frac{5\,\mathrm{i}\,\sqrt{e}\left(-\frac{\mathrm{ArcCsc}[c\,x]}{-\mathrm{i}\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{\mathrm{i}\left(\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(-\mathrm{i}\,c\,\sqrt{d}-\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}+\mathrm{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right)}{16\,d^{5/2}} - \frac{5\,\mathrm{i}\,\sqrt{e}\left(-\frac{\mathrm{ArcCsc}[c\,x]}{\mathrm{i}\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{\mathrm{i}\left(\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\mathrm{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\left(-\sqrt{e}+c\left(-\mathrm{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d-e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d-e}\left(\sqrt{d}-\mathrm{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d-e}}\right)}{\sqrt{d}}\right)}{16\,d^{5/2}} + \frac{1}{16\,d^2}$$

$$\sqrt{e}\left(\frac{\mathrm{i}\,c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\left(c^2\,d+e\right)\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsc}[c\,x]}{\sqrt{e}\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{\mathrm{i}\left(2\,c^2\,d+e\right)\mathrm{Log}\left[\frac{4\,d\,\sqrt{e}\,\sqrt{c^2\,d+e}\left(\mathrm{i}\,\sqrt{e}+c\left(c\,\sqrt{d}-\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\left(2\,c^2\,d+e\right)\left(-\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{d\left(c^2\,d+e\right)^{3/2}}\right) +$$

$$\frac{1}{16\,d^2}\sqrt{e}$$

$$\left(-\frac{\mathrm{i}\,c\,\sqrt{e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\left(c^2\,d+e\right)\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\mathrm{ArcCsc}[c\,x]}{\sqrt{e}\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\mathrm{ArcSin}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{\mathrm{i}\left(2\,c^2\,d+e\right)\mathrm{Log}\left[-\frac{4\,d\,\sqrt{e}\,\sqrt{c^2\,d+e}\left(-\mathrm{i}\,\sqrt{e}+c\left(c\,\sqrt{d}+\sqrt{c^2\,d+e}\,\sqrt{1-\frac{1}{c^2\,x^2}}\right)x\right)}{\left(2\,c^2\,d+e\right)\left(\mathrm{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{d\left(c^2\,d+e\right)^{3/2}}\right) -$$

$$\frac{1}{16\,d^3}\,i\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(-i\,c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)-$$

$$\frac{1}{16\,d^3}\,i\left(\pi^2-4\,\pi\,\text{ArcCsc}[c\,x]+8\,\text{ArcCsc}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(i\,c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\text{ArcCsc}[c\,x]\right)\right]}{\sqrt{c^2\,d+e}}\right]+\right.$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{i\,\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,i\,\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]+8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}-\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\frac{\left(\sqrt{e}+\sqrt{c^2\,d+e}\right)\,e^{-i\,\text{ArcCsc}[c\,x]}}{c\,\sqrt{d}}\right]+4\,\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)+$$

$$\left.\frac{-\text{ArcCsc}[c\,x]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[c\,x]}\right]+\frac{1}{2}\,i\,\left(\text{ArcCsc}[c\,x]^2+\text{PolyLog}\left[2,e^{2\,i\,\text{ArcCsc}[c\,x]}\right]\right)}{d^3}\right)$$

Problem 118: Result unnecessarily involves higher level functions.

$$\int x^5\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)\,dx$$

Optimal (type 3, 403 leaves, 12 steps):

$$-\frac{b\left(23 c^4 d^2+12 c^2 d e-75 e^2\right) x \sqrt{-1+c^2 x^2}\, \sqrt{d+e x^2}}{1680 c^5 e^2 \sqrt{c^2 x^2}}-\frac{b\left(29 c^2 d-25 e\right) x \sqrt{-1+c^2 x^2}\, \left(d+e x^2\right)^{3/2}}{840 c^3 e^2 \sqrt{c^2 x^2}}+$$
$$\frac{b x \sqrt{-1+c^2 x^2}\, \left(d+e x^2\right)^{5/2}}{42 c e^2 \sqrt{c^2 x^2}}+\frac{d^2\left(d+e x^2\right)^{3/2}(a+b\,\text{ArcCsc}[c x])}{3 e^3}-\frac{2 d\left(d+e x^2\right)^{5/2}(a+b\,\text{ArcCsc}[c x])}{5 e^3}+$$
$$\frac{\left(d+e x^2\right)^{7/2}(a+b\,\text{ArcCsc}[c x])}{7 e^3}-\frac{8 b c d^{7/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\right]}{105 e^3 \sqrt{c^2 x^2}}+\frac{b\left(105 c^6 d^3-35 c^4 d^2 e+63 c^2 d e^2+75 e^3\right) x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{1680 c^6 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 705 leaves):

$$\left(b d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3\left(\left(105 c^6 d^3-35 c^4 d^2 e+63 c^2 d e^2+75 e^3\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$
$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]$$
$$\left(\left(35 c^6 d^2 e^2 x^2-63 c^4 d e^3 x^2-75 c^2 e^4 x^2+c^8 d^3\left(128 d-105 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.32 c^8 d^3 x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(840 c^5 e^2\left(-1+c^2 x^2\right) \sqrt{d+e x^2}\left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-\right.\right.$$
$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left.\left(4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{1680 c^5 e^3}\sqrt{d+e x^2}\left(16 a c^5\left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right)+b e \sqrt{1-\frac{1}{c^2 x^2}}\, x\left(75 e^2+2 c^2 e\left(19 d+25 e x^2\right)+c^4\left(-41 d^2+22 d e x^2+40 e^2 x^4\right)\right)+\right.$$
$$\left.16 b c^5\left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right) \text{ArcCsc}[c x]\right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{d+e x^2}\, (a+b\,\text{ArcCsc}[c\,x])\, dx$$

Optimal (type 3, 294 leaves, 11 steps):

$$\frac{b\left(c^2 d+9 e\right) x \sqrt{-1+c^2 x^2}\, \sqrt{d+e x^2}}{120\, c^3\, e \sqrt{c^2 x^2}}+\frac{b\, x \sqrt{-1+c^2 x^2}\, \left(d+e x^2\right)^{3/2}}{20\, c\, e \sqrt{c^2 x^2}}-\frac{d\left(d+e x^2\right)^{3/2} (a+b\,\text{ArcCsc}[c\,x])}{3\, e^2}+$$
$$\frac{\left(d+e x^2\right)^{5/2} (a+b\,\text{ArcCsc}[c\,x])}{5\, e^2}+\frac{2\, b\, c\, d^{5/2}\, x\, \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\, \sqrt{-1+c^2 x^2}}\right]}{15\, e^2 \sqrt{c^2 x^2}}-\frac{b\left(15 c^4 d^2-10 c^2 d e-9 e^2\right) x\, \text{ArcTanh}\left[\frac{\sqrt{e}\, \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120\, c^4\, e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 627 leaves):

$$-\left(\left(b\, d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \left(\left(15 c^4 d^2-10 c^2 d e-9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.\right.$$
$$\left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$
$$4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\left(\left(10 c^4 d e^2 x^2+9 c^2 e^3 x^2+c^6 d^2 \left(16 d-15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.4 c^6 d^2 x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(60\, c^3\, e \left(-1+c^2 x^2\right) \sqrt{d+e x^2}\right.$$
$$\left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left.\left(4\, d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(-e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{120\, c^3\, e^2} \sqrt{d+e x^2}\, \left(8\, a\, c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right)+b\, e \sqrt{1-\frac{1}{c^2 x^2}}\, x \left(9 e+c^2 \left(7 d+6 e x^2\right)\right)+\right.$$
$$\left.8\, b\, c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right) \text{ArcCsc}[c\,x]\right)$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \sqrt{d+e x^2}\,(a+b \operatorname{ArcCsc}[c x])\,dx$$

Optimal (type 3, 195 leaves, 9 steps):

$$\frac{b x \sqrt{-1+c^2 x^2}\,\sqrt{d+e x^2}}{6 c \sqrt{c^2 x^2}}+\frac{\left(d+e x^2\right)^{3/2} (a+b \operatorname{ArcCsc}[c x])}{3 e}-\frac{b c d^{3/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 e \sqrt{c^2 x^2}}+\frac{b \left(3 c^2 d+e\right) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{6 c^2 \sqrt{e} \sqrt{c^2 x^2}}$$

Result (type 6, 547 leaves):

$$\Bigg(b d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \Bigg(\left(3 c^2 d+e\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$
$$2 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \Bigg(\left(-2 c^2 e^2 x^2+2 c^4 d \left(2 d-3 e x^2\right)\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+$$
$$c^4 d x^2 \left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(3 c \left(-1+c^2 x^2\right) \sqrt{d+e x^2}$$
$$\left(-4 c^2 e x^2 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left(4 d \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\Bigg)+$$
$$\frac{\sqrt{d+e x^2}\left(b e \sqrt{1-\frac{1}{c^2 x^2}}\, x+2 a c \left(d+e x^2\right)+2 b c \left(d+e x^2\right) \operatorname{ArcCsc}[c x]\right)}{6 c e}$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d+e x^2}\,(a+b \operatorname{ArcCsc}[c x])}{x^4}\,dx$$

Optimal (type 4, 328 leaves, 11 steps):

$$-\frac{2bc\left(c^2d+2e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3dx^3}+\frac{2bc^2\left(c^2d+2e\right)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2d}\right]}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}-\frac{b\left(c^2d+e\right)\left(2c^2d+3e\right)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2d}\right]}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Result (type 4, 247 leaves) :

$$-\frac{\sqrt{d+ex^2}\left(3a\left(d+ex^2\right)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\left(d+2c^2dx^2+4ex^2\right)+3b\left(d+ex^2\right)\text{ArcCsc}[c\,x]\right)}{9dx^3}+\left(i\,bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d\left(c^2d+2e\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\,x\right],-\frac{e}{c^2d}\right]-\left(2c^4d^2+5c^2de+3e^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\,x\right],-\frac{e}{c^2d}\right]\right)\right)\Big/\left(9\sqrt{-c^2}\,d\sqrt{1-c^2x^2}\sqrt{d+ex^2}\right)$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{d+ex^2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{x^6}\,dx$$

Optimal (type 4, 453 leaves, 12 steps) :

$$-\frac{b c \left(24 c^4 d^2+19 c^2 d e-31 e^2\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{225 d^2 \sqrt{c^2 x^2}}-\frac{b c \left(12 c^2 d-e\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{225 d x^2 \sqrt{c^2 x^2}}-$$
$$\frac{b c \sqrt{-1+c^2 x^2} \left(d+e x^2\right)^{3/2}}{25 d x^4 \sqrt{c^2 x^2}}-\frac{\left(d+e x^2\right)^{3/2} (a+b \, \text{ArcCsc}[c x])}{5 d x^5}+\frac{2 e \left(d+e x^2\right)^{3/2} (a+b \, \text{ArcCsc}[c x])}{15 d^2 x^3}+$$
$$\frac{b c^2 \left(24 c^4 d^2+19 c^2 d e-31 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \, \text{EllipticE}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{225 d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}}-$$
$$\frac{b \left(c^2 d+e\right) \left(24 c^4 d^2+7 c^2 d e-30 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \, \text{EllipticF}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{225 d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}$$

Result (type 4, 325 leaves):

$$-\frac{1}{225 d^2 x^5}\sqrt{d+e x^2} \left(15 a \left(3 d^2+d e x^2-2 e^2 x^4\right)+\right.$$
$$\left. b c \sqrt{1-\frac{1}{c^2 x^2}} \, x \left(-31 e^2 x^4+d e x^2 \left(8+19 c^2 x^2\right)+3 d^2 \left(3+4 c^2 x^2+8 c^4 x^4\right)\right)+15 b \left(3 d^2+d e x^2-2 e^2 x^4\right) \text{ArcCsc}[c x]\right)+$$
$$\left(i b c \sqrt{1-\frac{1}{c^2 x^2}} \, x \sqrt{1+\frac{e x^2}{d}} \left(c^2 d \left(24 c^4 d^2+19 c^2 d e-31 e^2\right) \text{EllipticE}\left[i \, \text{ArcSinh}\left[\sqrt{-c^2} \, x\right],-\frac{e}{c^2 d}\right]+\right.\right.$$
$$\left.\left.\left(-24 c^6 d^3-31 c^4 d^2 e+23 c^2 d e^2+30 e^3\right) \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{-c^2} \, x\right],-\frac{e}{c^2 d}\right]\right)\right) \Big/ \left(225 \sqrt{-c^2} \, d^2 \sqrt{1-c^2 x^2} \sqrt{d+e x^2}\right)$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int x^3 \left(d+e x^2\right)^{3/2} (a+b \, \text{ArcCsc}[c x]) \, dx$$

Optimal (type 3, 374 leaves, 12 steps):

$$-\frac{b\left(3 c^4 d^2-38 c^2 d e-25 e^2\right) x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{560 c^5 e \sqrt{c^2 x^2}}+\frac{b\left(13 c^2 d+25 e\right) x \sqrt{-1+c^2 x^2}\left(d+e x^2\right)^{3/2}}{840 c^3 e \sqrt{c^2 x^2}}+$$
$$\frac{b x \sqrt{-1+c^2 x^2}\left(d+e x^2\right)^{5/2}}{42 c e \sqrt{c^2 x^2}}-\frac{d\left(d+e x^2\right)^{5/2}(a+b \operatorname{ArcCsc}[c x])}{5 e^2}+\frac{\left(d+e x^2\right)^{7/2}(a+b \operatorname{ArcCsc}[c x])}{7 e^2}+$$
$$\frac{2 b c d^{7/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{35 e^2 \sqrt{c^2 x^2}}-\frac{b\left(35 c^6 d^3-35 c^4 d^2 e-63 c^2 d e^2-25 e^3\right) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{560 c^6 e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 679 leaves) :

$$-\left(\left(b d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3\left(\left(35 c^6 d^3-35 c^4 d^2 e-63 c^2 d e^2-25 e^3\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.\right.$$
$$\left(c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]$$
$$\left(\left(35 c^6 d^2 e^2 x^2+63 c^4 d e^3 x^2+25 c^2 e^4 x^2+c^8 d^3\left(32 d-35 e x^2\right)\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.8 c^8 d^3 x^2\left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(280 c^5 e\left(-1+c^2 x^2\right) \sqrt{d+e x^2}\left(-4 c^2 e x^2 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-\right.\right.$$
$$\left.e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left.\left(4 d \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{1680 c^5 e^2} \sqrt{d+e x^2}\left(-48 a c^5\left(2 d-5 e x^2\right)\left(d+e x^2\right)^2+b e \sqrt{1-\frac{1}{c^2 x^2}}\, x\left(75 e^2+2 c^2 e\left(82 d+25 e x^2\right)+c^4\left(57 d^2+106 d e x^2+40 e^2 x^4\right)\right)-\right.$$
$$\left.48 b c^5\left(2 d-5 e x^2\right)\left(d+e x^2\right)^2 \operatorname{ArcCsc}[c x]\right)$$

Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x\left(d+e x^2\right)^{3/2}(a+b\,\text{ArcCsc}[c\,x])\,dx$$

Optimal (type 3, 262 leaves, 10 steps):

$$\frac{b\left(7 c^2 d+3 e\right) x \sqrt{-1+c^2 x^2}\,\sqrt{d+e x^2}}{40 c^3 \sqrt{c^2 x^2}}+\frac{b\,x \sqrt{-1+c^2 x^2}\,\left(d+e x^2\right)^{3/2}}{20 c \sqrt{c^2 x^2}}+$$

$$\frac{\left(d+e x^2\right)^{5/2}(a+b\,\text{ArcCsc}[c\,x])}{5 e}-\frac{b\,c\,d^{5/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\right]}{5 e \sqrt{c^2 x^2}}+\frac{b\left(15 c^4 d^2+10 c^2 d e+3 e^2\right) x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{40 c^4 \sqrt{e}\,\sqrt{c^2 x^2}}$$

Result (type 6, 602 leaves):

$$\left(b\,d\sqrt{1-\frac{1}{c^2 x^2}}\,x^3\left(\left(15 c^4 d^2+10 c^2 d e+3 e^2\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$

$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\left(\left(-10 c^4 d e^2 x^2-3 c^2 e^3 x^2+c^6 d^2\left(8 d-15 e x^2\right)\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$

$$\left.\left.\left.2 c^6 d^2 x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(20 c^3\left(-1+c^2 x^2\right)\sqrt{d+e x^2}\left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-\right.\right.$$

$$\left.e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$

$$\left.\left(4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$

$$\frac{\sqrt{d+e x^2}\left(8 a c^3\left(d+e x^2\right)^2+b\,e\sqrt{1-\frac{1}{c^2 x^2}}\,x\left(3 e+c^2\left(9 d+2 e x^2\right)\right)+8 b c^3\left(d+e x^2\right)^2\text{ArcCsc}[c\,x]\right)}{40 c^3 e}$$

Problem 136: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^{3/2}(a+b\,\text{ArcCsc}[c\,x])}{x^6}\,dx$$

Optimal (type 4, 416 leaves, 12 steps):

$$-\frac{b c \left(8 c^4 d^2+23 c^2 d e+23 e^2\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 d \sqrt{c^2 x^2}}-\frac{4 b c \left(c^2 d+2 e\right) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 x^2 \sqrt{c^2 x^2}}-\frac{b c \sqrt{-1+c^2 x^2} \left(d+e x^2\right)^{3/2}}{25 x^4 \sqrt{c^2 x^2}}-$$

$$\frac{\left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsc}[c x]\right)}{5 d x^5}+\frac{b c^2 \left(8 c^4 d^2+23 c^2 d e+23 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \text{ EllipticE}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{75 d \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}}-$$

$$\frac{b \left(c^2 d+e\right) \left(8 c^4 d^2+19 c^2 d e+15 e^2\right) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \text{ EllipticF}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{75 d \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}$$

Result (type 4, 303 leaves) :

$$-\frac{1}{75 d x^5}\sqrt{d+e x^2} \left(15 a \left(d+e x^2\right)^2+b c \sqrt{1-\frac{1}{c^2 x^2}}\; x \left(23 e^2 x^4+d e x^2 \left(11+23 c^2 x^2\right)+d^2 \left(3+4 c^2 x^2+8 c^4 x^4\right)\right)+15 b \left(d+e x^2\right)^2 \text{ArcCsc}[c x]\right)+$$

$$\left(\text{i} b c \sqrt{1-\frac{1}{c^2 x^2}}\; x \sqrt{1+\frac{e x^2}{d}} \left(c^2 d \left(8 c^4 d^2+23 c^2 d e+23 e^2\right) \text{EllipticE}\left[\text{i} \text{ ArcSinh}\left[\sqrt{-c^2}\; x\right],-\frac{e}{c^2 d}\right]-\right.\right.$$

$$\left.\left.\left(8 c^6 d^3+27 c^4 d^2 e+34 c^2 d e^2+15 e^3\right) \text{EllipticF}\left[\text{i} \text{ ArcSinh}\left[\sqrt{-c^2}\; x\right],-\frac{e}{c^2 d}\right]\right)\right) \Big/ \left(75 \sqrt{-c^2}\; d \sqrt{1-c^2 x^2} \sqrt{d+e x^2}\right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsc}[c x]\right)}{x^8} \, dx$$

Optimal (type 4, 554 leaves, 13 steps) :

$$-\frac{b\,c\,\left(240\,c^6\,d^3+528\,c^4\,d^2\,e+193\,c^2\,d\,e^2-247\,e^3\right)\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{3675\,d^2\,\sqrt{c^2\,x^2}}-\frac{b\,c\,\left(120\,c^4\,d^2+159\,c^2\,d\,e-37\,e^2\right)\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}}{3675\,d\,x^2\,\sqrt{c^2\,x^2}}-$$

$$\frac{b\,c\,\left(30\,c^2\,d+11\,e\right)\sqrt{-1+c^2\,x^2}\,\left(d+e\,x^2\right)^{3/2}}{1225\,d\,x^4\,\sqrt{c^2\,x^2}}-\frac{b\,c\,\sqrt{-1+c^2\,x^2}\,\left(d+e\,x^2\right)^{5/2}}{49\,d\,x^6\,\sqrt{c^2\,x^2}}-\frac{\left(d+e\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{7\,d\,x^7}+$$

$$\frac{2\,e\,\left(d+e\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{35\,d^2\,x^5}+\frac{b\,c^2\,\left(240\,c^6\,d^3+528\,c^4\,d^2\,e+193\,c^2\,d\,e^2-247\,e^3\right)x\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]}{3675\,d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}}-$$

$$\left(2\,b\,\left(c^2\,d+e\right)\left(120\,c^6\,d^3+204\,c^4\,d^2\,e+17\,c^2\,d\,e^2-105\,e^3\right)x\,\sqrt{1-c^2\,x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2\,d}\right]\right)\Big/$$

$$\left(3675\,d^2\,\sqrt{c^2\,x^2}\,\sqrt{-1+c^2\,x^2}\,\sqrt{d+e\,x^2}\right)$$

Result (type 4, 383 leaves):

$$-\frac{1}{3675\,d^2\,x^7}\sqrt{d+e\,x^2}\,\Bigg(105\,a\,\left(5\,d-2\,e\,x^2\right)\left(d+e\,x^2\right)^2+$$

$$b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\left(-247\,e^3\,x^6+d\,e^2\,x^4\,\left(71+193\,c^2\,x^2\right)+3\,d^2\,e\,x^2\,\left(61+83\,c^2\,x^2+176\,c^4\,x^4\right)+15\,d^3\,\left(5+6\,c^2\,x^2+8\,c^4\,x^4+16\,c^6\,x^6\right)\right)+$$

$$105\,b\,\left(5\,d-2\,e\,x^2\right)\left(d+e\,x^2\right)^2\,\text{ArcCsc}[c\,x]\Bigg)+$$

$$\Bigg(i\,b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\sqrt{1+\frac{e\,x^2}{d}}\,\Bigg(c^2\,d\,\left(240\,c^6\,d^3+528\,c^4\,d^2\,e+193\,c^2\,d\,e^2-247\,e^3\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\;x\right],-\frac{e}{c^2\,d}\right]-$$

$$2\,\left(120\,c^8\,d^4+324\,c^6\,d^3\,e+221\,c^4\,d^2\,e^2-88\,c^2\,d\,e^3-105\,e^4\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-c^2}\;x\right],-\frac{e}{c^2\,d}\right]\Bigg)\Bigg)\Big/\left(3675\,\sqrt{-c^2}\;d^2\,\sqrt{1-c^2\,x^2}\,\sqrt{d+e\,x^2}\right)$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int\frac{x^5\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 3, 321 leaves, 11 steps):

$$-\frac{b\left(19 c^2 d-9 e\right) x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{120 c^3 e^2 \sqrt{c^2 x^2}}+\frac{b x \sqrt{-1+c^2 x^2}\left(d+e x^2\right)^{3/2}}{20 c e^2 \sqrt{c^2 x^2}}+\frac{d^2 \sqrt{d+e x^2}\left(a+b \operatorname{ArcCsc}[c x]\right)}{e^3}-\frac{2 d\left(d+e x^2\right)^{3/2}\left(a+b \operatorname{ArcCsc}[c x]\right)}{3 e^3}+\frac{\left(d+e x^2\right)^{5/2}\left(a+b \operatorname{ArcCsc}[c x]\right)}{5 e^3}-\frac{8 b c d^{5/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{15 e^3 \sqrt{c^2 x^2}}+\frac{b\left(45 c^4 d^2-10 c^2 d e+9 e^2\right) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120 c^4 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 629 leaves):

$$\Bigg(b d \sqrt{1-\frac{1}{c^2 x^2}}\, x^3 \Bigg(\left(45 c^4 d^2-10 c^2 d e+9 e^2\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]-e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+ 4 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \Bigg(\left(10 c^4 d e^2 x^2-9 c^2 e^3 x^2+c^6 d^2\left(64 d-45 e x^2\right)\right) \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+ 16 c^6 d^2 x^2\left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\Bigg)\Bigg)\Bigg) \Bigg/ \Bigg(60 c^3 e^2\left(-1+c^2 x^2\right) \sqrt{d+e x^2}\left(-4 c^2 e x^2 \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]- e \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(4 d \operatorname{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(-e \operatorname{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \operatorname{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\Bigg)+ \frac{1}{120 c^3 e^3} \sqrt{d+e x^2}\left(8 a c^3\left(8 d^2-4 d e x^2+3 e^2 x^4\right)+b e \sqrt{1-\frac{1}{c^2 x^2}}\, x\left(9 e+c^2\left(-13 d+6 e x^2\right)\right)+8 b c^3\left(8 d^2-4 d e x^2+3 e^2 x^4\right) \operatorname{ArcCsc}[c x]\right)$$

Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3\left(a+b \operatorname{ArcCsc}[c x]\right)}{\sqrt{d+e x^2}}\, dx$$

Optimal (type 3, 225 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{-1+c^2x^2}\,\sqrt{d+e\,x^2}}{6\,c\,e\,\sqrt{c^2x^2}}-\frac{d\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}+$$

$$\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsc}[c\,x]\right)}{3\,e^2}+\frac{2\,b\,c\,d^{3/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{3\,e^2\,\sqrt{c^2x^2}}-\frac{b\left(3\,c^2d-e\right)x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{6\,c^2\,e^{3/2}\,\sqrt{c^2x^2}}$$

Result (type 6, 554 leaves):

$$-\left(\left(b\,d\,\sqrt{1-\frac{1}{c^2x^2}}\,x^3\right.\right.$$

$$\left(\left(3\,c^2d-e\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]\left(c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)+\right.$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\left(\left(c^2e^2x^2+c^4d\left(4\,d-3\,e\,x^2\right)\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+\right.$$

$$\left.\left.\left.c^4d\,x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/\left(3\,c\,e\left(-1+c^2x^2\right)\sqrt{d+e\,x^2}\right.$$

$$\left(-4\,c^2e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+c^2d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)$$

$$\left.\left.\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e\,x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]+c^2d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{\sqrt{d+e\,x^2}\left(-4\,a\,c\,d+b\,e\,\sqrt{1-\frac{1}{c^2x^2}}\,x+2\,a\,c\,e\,x^2+2\,b\,c\left(-2\,d+e\,x^2\right)\text{ArcCsc}[c\,x]\right)}{6\,c\,e^2}$$

Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\sqrt{d+e\,x^2}}\,dx$$

Optimal (type 3, 132 leaves, 9 steps):

$$\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{e}-\frac{b c \sqrt{d}\, x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\sqrt{-1+c^2 x^2}}\right]}{e\sqrt{c^2 x^2}}+\frac{b x\,\text{ArcTanh}\left[\frac{\sqrt{e}\sqrt{-1+c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{\sqrt{e}\sqrt{c^2 x^2}}$$

Result (type 6, 271 leaves):

$$-\left(\left(3 b\left(c^2 d+e\right)\sqrt{1-\frac{1}{c^2 x^2}}\sqrt{d+e x^2}\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]\right)\Big/\left(c e x\left(-3\left(c^2 d+e\right)\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]+\left(-1+c^2 x^2\right)\left(2\left(c^2 d+e\right)\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]-e\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},\frac{e-c^2 e x^2}{c^2 d+e},1-c^2 x^2\right]\right)\right)\right)\right)+\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{e}$$

Problem 146: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsc}[c x]}{x^4\sqrt{d+e x^2}}\,dx$$

Optimal (type 4, 362 leaves, 11 steps):

$$-\frac{b c\left(2 c^2 d-5 e\right)\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}}{9 d^2\sqrt{c^2 x^2}}-\frac{b c\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}}{9 d x^2\sqrt{c^2 x^2}}-\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{3 d x^3}+\frac{2 e\sqrt{d+e x^2}\,(a+b\,\text{ArcCsc}[c x])}{3 d^2 x}+\frac{b c^2\left(2 c^2 d-5 e\right) x\sqrt{1-c^2 x^2}\sqrt{d+e x^2}\,\text{EllipticE}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{9 d^2\sqrt{c^2 x^2}\sqrt{-1+c^2 x^2}\sqrt{1+\frac{e x^2}{d}}}-\frac{2 b\left(c^2 d-3 e\right)\left(c^2 d+e\right) x\sqrt{1-c^2 x^2}\sqrt{1+\frac{e x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{9 d^2\sqrt{c^2 x^2}\sqrt{-1+c^2 x^2}\sqrt{d+e x^2}}$$

Result (type 4, 249 leaves):

$$-\frac{\sqrt{d+e x^2}\left(b c \sqrt{1-\frac{1}{c^2 x^2}}\, x\left(d+2 c^2 d x^2-5 e x^2\right)+3 a\left(d-2 e x^2\right)+3 b\left(d-2 e x^2\right) \text{ArcCsc}[c x]\right)}{9 d^2 x^3}+$$

$$\left(\mathrm{i}\, b c \sqrt{1-\frac{1}{c^2 x^2}}\, x \sqrt{1+\frac{e x^2}{d}}\left(c^2 d\left(2 c^2 d-5 e\right) \text{EllipticE}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{-c^2}\, x\right],-\frac{e}{c^2 d}\right]+\right.\right.$$

$$\left.\left.2\left(-c^4 d^2+2 c^2 d e+3 e^2\right) \text{EllipticF}\left[\mathrm{i}\, \text{ArcSinh}\left[\sqrt{-c^2}\, x\right],-\frac{e}{c^2 d}\right]\right)\right) \Big/ \left(9 \sqrt{-c^2}\, d^2 \sqrt{1-c^2 x^2} \sqrt{d+e x^2}\right)$$

Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^5\left(a+b \text{ArcCsc}[c x]\right)}{\left(d+e x^2\right)^{3/2}}\, dx$$

Optimal (type 3, 252 leaves, 10 steps):

$$\frac{b x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{6 c e^2 \sqrt{c^2 x^2}}-\frac{d^2\left(a+b \text{ArcCsc}[c x]\right)}{e^3 \sqrt{d+e x^2}}-\frac{2 d \sqrt{d+e x^2}\left(a+b \text{ArcCsc}[c x]\right)}{e^3}+$$

$$\frac{\left(d+e x^2\right)^{3/2}\left(a+b \text{ArcCsc}[c x]\right)}{3 e^3}+\frac{8 b c d^{3/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 e^3 \sqrt{c^2 x^2}}-\frac{b\left(9 c^2 d-e\right) x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{6 c^2 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 586 leaves):

$$-\Bigg(\Bigg(\Bigg(b\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x^3$$

$$\left(\left(9\,c^2\,d-e\right)\,\text{AppellF1}\left[1,\,\frac{1}{2},\,\frac{1}{2},\,2,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]\,\left(c^2\,d\,\text{AppellF1}\left[2,\,\frac{1}{2},\,\frac{3}{2},\,3,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\,\frac{3}{2},\,\frac{1}{2},\,3,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]\right)+\right.$$

$$4\,\text{AppellF1}\left[1,\,\frac{1}{2},\,\frac{1}{2},\,2,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]\,\Bigg(\left(c^2\,e^2\,x^2+c^4\,d\,\left(16\,d-9\,e\,x^2\right)\right)\,\text{AppellF1}\left[1,\,\frac{1}{2},\,\frac{1}{2},\,2,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]+$$

$$\left.4\,c^4\,d\,x^2\,\left(-e\,\text{AppellF1}\left[2,\,\frac{1}{2},\,\frac{3}{2},\,3,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\,\frac{3}{2},\,\frac{1}{2},\,3,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]\right)\Bigg)\right)\Bigg)\Bigg/\Bigg(3\,c\,e^2\,\left(-1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}$$

$$\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\,\frac{1}{2},\,\frac{1}{2},\,2,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]+c^2\,d\,\text{AppellF1}\left[2,\,\frac{1}{2},\,\frac{3}{2},\,3,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\,\frac{3}{2},\,\frac{1}{2},\,3,\,\frac{1}{c^2\,x^2},\,-\frac{d}{e\,x^2}\right]\right)$$

$$\left(4\,d\,\text{AppellF1}\left[1,\,\frac{1}{2},\,\frac{1}{2},\,2,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]+x^2\,\left(-e\,\text{AppellF1}\left[2,\,\frac{1}{2},\,\frac{3}{2},\,3,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\,\frac{3}{2},\,\frac{1}{2},\,3,\,c^2\,x^2,\,-\frac{e\,x^2}{d}\right]\right)\right)\Bigg)\Bigg)+$$

$$\frac{b\,e\,\sqrt{1-\frac{1}{c^2\,x^2}}\;x\,\left(d+e\,x^2\right)-2\,a\,c\,\left(8\,d^2+4\,d\,e\,x^2-e^2\,x^4\right)-2\,b\,c\,\left(8\,d^2+4\,d\,e\,x^2-e^2\,x^4\right)\,\text{ArcCsc}[c\,x]}{6\,c\,e^3\,\sqrt{d+e\,x^2}}$$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 156 leaves, 9 steps):

$$\frac{d\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2\,\sqrt{d+e\,x^2}}+\frac{\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2}-\frac{2\,b\,c\,\sqrt{d}\;x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2\,x^2}}\right]}{e^2\,\sqrt{c^2\,x^2}}+\frac{b\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{e^{3/2}\,\sqrt{c^2\,x^2}}$$

Result (type 6, 326 leaves):

$$\frac{1}{e\left(-1+c^2x^2\right)\sqrt{d+e x^2}}2\,b\,c\,d\sqrt{1-\frac{1}{c^2x^2}}\;x^3\left(-\left(\left(2\,c^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e x^2}\right]\right)\Big/\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e x^2}\right]\right)\right)+\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e x^2}{d}\right]\Big/\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{e x^2}{d}\right]+x^2\left(-e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,c^2x^2,-\frac{e x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,c^2x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+\frac{\left(2\,d+e\,x^2\right)\left(a+b\,\text{ArcCsc}[c\,x]\right)}{e^2\sqrt{d+e\,x^2}}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x\left(a+b\,\text{ArcCsc}[c\,x]\right)}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$-\frac{a+b\,\text{ArcCsc}[c\,x]}{e\sqrt{d+e\,x^2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{\sqrt{d}\,e\sqrt{c^2x^2}}$$

Result (type 6, 190 leaves):

$$\left(2\,b\,c^3\sqrt{1-\frac{1}{c^2x^2}}\;x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(\left(-1+c^2x^2\right)\sqrt{d+e\,x^2}\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\right)-\frac{a+b\,\text{ArcCsc}[c\,x]}{e\sqrt{d+e\,x^2}}$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcCsc}[c\,x]}{x^2\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 4, 275 leaves, 10 steps):

$$-\frac{b c \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{d^2 \sqrt{c^2 x^2}}-\frac{a+b \text{ArcCsc}[c x]}{d x \sqrt{d+e x^2}}-\frac{2 e x (a+b \text{ArcCsc}[c x])}{d^2 \sqrt{d+e x^2}}+$$
$$\frac{b c^2 x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \text{EllipticE}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}}-\frac{b (c^2 d+2 e) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \text{EllipticF}\left[\text{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}$$

Result (type 4, 213 leaves):

$$\frac{-b c \sqrt{1-\frac{1}{c^2 x^2}} x (d+e x^2)-a (d+2 e x^2)-b (d+2 e x^2) \text{ArcCsc}[c x]}{d^2 x \sqrt{d+e x^2}}+$$
$$\left(i b c \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{1+\frac{e x^2}{d}} \left(c^2 d \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-c^2} x\right],-\frac{e}{c^2 d}\right]-(c^2 d+2 e) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-c^2} x\right],-\frac{e}{c^2 d}\right]\right)\right)\Big/$$
$$\left(\sqrt{-c^2} d^2 \sqrt{1-c^2 x^2} \sqrt{d+e x^2}\right)$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 (a+b \text{ArcCsc}[c x])}{(d+e x^2)^{5/2}} \, dx$$

Optimal (type 3, 243 leaves, 10 steps):

$$\frac{b c d x \sqrt{-1+c^2 x^2}}{3 e^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}}-\frac{d^2 (a+b \text{ArcCsc}[c x])}{3 e^3 (d+e x^2)^{3/2}}+\frac{2 d (a+b \text{ArcCsc}[c x])}{e^3 \sqrt{d+e x^2}}+$$
$$\frac{\sqrt{d+e x^2} (a+b \text{ArcCsc}[c x])}{e^3}-\frac{8 b c \sqrt{d} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 e^3 \sqrt{c^2 x^2}}+\frac{b x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 416 leaves):

$$\Bigg(2\,b\,c\,d\,\sqrt{1-\frac{1}{c^2 x^2}}\;x^3$$

$$\Bigg(-\Bigg(\Bigg(8\,c^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Bigg/\Bigg(4\,c^2\,e\,x^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]-c^2\,d\,\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]+$$

$$e\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Bigg)+\Bigg(3\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\Big]\Bigg)\Bigg/$$

$$\Bigg(4\,d\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,c^2 x^2,-\frac{e\,x^2}{d}\Big]+x^2\Bigg(-e\,\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\Big]+c^2\,d\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,c^2 x^2,-\frac{e\,x^2}{d}\Big]\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\Big(3\,e^2\,(-1+c^2 x^2)\,\sqrt{d+e\,x^2}\Big)+\Bigg(b\,c\,d\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\;x\,(d+e\,x^2)+a\,(c^2\,d+e)\,(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4)+$$

$$b\,(c^2\,d+e)\,(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4)\,\text{ArcCsc}[c\,x]\Bigg)\Bigg/\Big(3\,e^3\,(c^2\,d+e)\,(d+e\,x^2)^{3/2}\Big)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int\frac{x^3\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x^2)^{5/2}}\,dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$-\frac{b\,c\,x\,\sqrt{-1+c^2 x^2}}{3\,e\,(c^2\,d+e)\,\sqrt{c^2 x^2}\,\sqrt{d+e\,x^2}}+\frac{d\,(a+b\,\text{ArcCsc}[c\,x])}{3\,e^2\,(d+e\,x^2)^{3/2}}-\frac{a+b\,\text{ArcCsc}[c\,x]}{e^2\,\sqrt{d+e\,x^2}}+\frac{2\,b\,c\,x\,\text{ArcTan}\Big[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2 x^2}}\Big]}{3\,\sqrt{d}\,e^2\,\sqrt{c^2 x^2}}$$

Result (type 6, 270 leaves):

$$\Bigg(4\,b\,c^3\,\sqrt{1-\frac{1}{c^2 x^2}}\;x^3\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Bigg/\Bigg(3\,e\,(-1+c^2 x^2)\,\sqrt{d+e\,x^2}$$

$$\Bigg(4\,c^2\,e\,x^2\,\text{AppellF1}\Big[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]-c^2\,d\,\text{AppellF1}\Big[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]+e\,\text{AppellF1}\Big[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2 x^2},-\frac{d}{e\,x^2}\Big]\Bigg)\Bigg)+$$

$$\frac{-b\,c\,e\,\sqrt{1-\frac{1}{c^2 x^2}}\;x\,(d+e\,x^2)-a\,(c^2\,d+e)\,(2\,d+3\,e\,x^2)-b\,(c^2\,d+e)\,(2\,d+3\,e\,x^2)\,\text{ArcCsc}[c\,x]}{3\,e^2\,(c^2\,d+e)\,(d+e\,x^2)^{3/2}}$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int \frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{(d+e\,x^2)^{5/2}}\,dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$\frac{b\,c\,x\,\sqrt{-1+c^2x^2}}{3\,d\,(c^2d+e)\,\sqrt{c^2x^2}\,\sqrt{d+e\,x^2}}-\frac{a+b\,\text{ArcCsc}[c\,x]}{3\,e\,(d+e\,x^2)^{3/2}}+\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right]}{3\,d^{3/2}\,e\,\sqrt{c^2x^2}}$$

Result (type 6, 254 leaves):

$$\left(2\,b\,c^3\,\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(3\,d\,(-1+c^2x^2)\,\sqrt{d+e\,x^2}\right.$$
$$\left.\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,\frac{1}{c^2x^2},-\frac{d}{e\,x^2}\right]\right)\right)+$$
$$\frac{-a\,d\,(c^2d+e)+b\,c\,e\,\sqrt{1-\frac{1}{c^2x^2}}\,x\,(d+e\,x^2)-b\,d\,(c^2d+e)\,\text{ArcCsc}[c\,x]}{3\,d\,e\,(c^2d+e)\,(d+e\,x^2)^{3/2}}$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCsc}[c\,x]}{(d+e\,x^2)^{5/2}}\,dx$$

Optimal (type 4, 296 leaves, 10 steps):

$$-\frac{b\,c\,e\,x^2\,\sqrt{-1+c^2x^2}}{3\,d^2\,(c^2d+e)\,\sqrt{c^2x^2}\,\sqrt{d+e\,x^2}}+\frac{x\,(a+b\,\text{ArcCsc}[c\,x])}{3\,d\,(d+e\,x^2)^{3/2}}+\frac{2\,x\,(a+b\,\text{ArcCsc}[c\,x])}{3\,d^2\,\sqrt{d+e\,x^2}}+$$
$$\frac{b\,c^2\,x\,\sqrt{1-c^2x^2}\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2d}\right]}{3\,d^2\,(c^2d+e)\,\sqrt{c^2x^2}\,\sqrt{-1+c^2x^2}\,\sqrt{1+\frac{e\,x^2}{d}}}+\frac{2\,b\,x\,\sqrt{1-c^2x^2}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[\text{ArcSin}[c\,x],-\frac{e}{c^2d}\right]}{3\,d^2\,\sqrt{c^2x^2}\,\sqrt{-1+c^2x^2}\,\sqrt{d+e\,x^2}}$$

Result (type 4, 249 leaves):

$$\frac{x\left(-b\,c\,e\sqrt{1-\frac{1}{c^2 x^2}}\;x\left(d+e\,x^2\right)+a\left(c^2 d+e\right)\left(3\,d+2\,e\,x^2\right)+b\left(c^2 d+e\right)\left(3\,d+2\,e\,x^2\right)\text{ArcCsc}[c\,x]\right)}{3\,d^2\left(c^2 d+e\right)\left(d+e\,x^2\right)^{3/2}}+$$

$$\left(\mathbb{i}\,b\,c\sqrt{1-\frac{1}{c^2 x^2}}\;x\sqrt{1+\frac{e\,x^2}{d}}\left(c^2 d\,\text{EllipticE}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{-c^2}\;x\right],-\frac{e}{c^2 d}\right]+2\left(c^2 d+e\right)\text{EllipticF}\left[\mathbb{i}\,\text{ArcSinh}\left[\sqrt{-c^2}\;x\right],-\frac{e}{c^2 d}\right]\right)\right)\Big/$$
$$\left(3\sqrt{-c^2}\;d^2\left(c^2 d+e\right)\sqrt{1-c^2 x^2}\;\sqrt{d+e\,x^2}\right)$$

Test results for the 49 problems in "5.6.2 Inverse cosecant functions.m"

Problem 13: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{ArcCsc}\left[\frac{a}{x}\right]}{x^2}\,dx$$

Optimal (type 3, 32 leaves, 5 steps):

$$-\frac{\text{ArcSin}\left[\frac{x}{a}\right]}{x}-\frac{\text{ArcTanh}\left[\sqrt{1-\frac{x^2}{a^2}}\right]}{a}$$

Result (type 3, 93 leaves):

$$-\frac{\text{ArcCsc}\left[\frac{a}{x}\right]}{x}-\frac{\sqrt{-1+\frac{a^2}{x^2}}\;x\left(-\text{Log}\left[1-\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\;x}\right]+\text{Log}\left[1+\frac{a}{\sqrt{-1+\frac{a^2}{x^2}}\;x}\right]\right)}{2\,a^2\sqrt{1-\frac{x^2}{a^2}}}$$

Problem 16: Result unnecessarily involves higher level functions.

$$\int\frac{\text{ArcCsc}\left[a\,x^n\right]}{x}\,dx$$

Optimal (type 4, 69 leaves, 7 steps):

$$\frac{i\,\text{ArcCsc}[a\,x^n]^2}{2\,n}-\frac{\text{ArcCsc}[a\,x^n]\,\text{Log}\left[1-e^{2\,i\,\text{ArcCsc}[a\,x^n]}\right]}{n}+\frac{i\,\text{PolyLog}\left[2,\,e^{2\,i\,\text{ArcCsc}[a\,x^n]}\right]}{2\,n}$$

Result (type 5, 63 leaves) :

$$-\frac{x^{-n}\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},\frac{x^{-2n}}{a^2}\right]}{a\,n}+\left(\text{ArcCsc}[a\,x^n]-\text{ArcSin}\left[\frac{x^{-n}}{a}\right]\right)\text{Log}[x]$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \text{ArcCsc}[a+b\,x]\,dx$$

Optimal (type 3, 36 leaves, 5 steps) :

$$\frac{(a+b\,x)\,\text{ArcCsc}[a+b\,x]}{b}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{(a+b\,x)^2}}\right]}{b}$$

Result (type 3, 120 leaves) :

$$x\,\text{ArcCsc}[a+b\,x]+\frac{(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\left(a\,\text{ArcTan}\left[\frac{1}{\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}}\right]+\text{Log}\left[a+b\,x+\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}\right]\right)}{b\sqrt{-1+a^2+2\,a\,b\,x+b^2\,x^2}}$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCsc}[a+b\,x]}{x^2}\,dx$$

Optimal (type 3, 69 leaves, 6 steps) :

$$-\frac{b\,\text{ArcCsc}[a+b\,x]}{a}-\frac{\text{ArcCsc}[a+b\,x]}{x}-\frac{2\,b\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{a\sqrt{1-a^2}}$$

Result (type 3, 115 leaves) :

$$-\frac{\text{ArcCsc}[a+b\,x]}{x}+\frac{b\left(-\text{ArcSin}\left[\frac{1}{a+b\,x}\right]+\frac{i\,\text{Log}\left[\frac{2\left(-\frac{i\,a\,(-1+a^2+a\,b\,x)}{\sqrt{1-a^2}}-a\,(a+b\,x)\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{b\,x}\right]}{\sqrt{1-a^2}}\right)}{a}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCsc}[a+b x]}{x^3} \, dx$$

Optimal (type 3, 123 leaves, 8 steps):

$$-\frac{b\,(a+b x)\,\sqrt{1-\frac{1}{(a+b x)^2}}}{2 a\,\left(1-a^2\right)\,x}+\frac{b^2\,\text{ArcCsc}[a+b x]}{2 a^2}-\frac{\text{ArcCsc}[a+b x]}{2 x^2}+\frac{\left(1-2 a^2\right)\,b^2\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b x]\right]}{\sqrt{1-a^2}}\right]}{a^2\,\left(1-a^2\right)^{3/2}}$$

Result (type 3, 199 leaves):

$$\frac{1}{2 x^2}\left(\frac{b x\,(a+b x)\,\sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}}{a\,\left(-1+a^2\right)}-\text{ArcCsc}[a+b x]+\frac{b^2 x^2\,\text{ArcSin}\left[\frac{1}{a+b x}\right]}{a^2}+\frac{i\,\left(-1+2 a^2\right)\,b^2 x^2\,\text{Log}\left[\frac{4\,(-1+a)\,a^2\,(1+a)\,\left(\frac{i\,\left(-1+a^2+a b x\right)}{\sqrt{1-a^2}}+(a+b x)\,\sqrt{\frac{-1+a^2+2 a b x+b^2 x^2}{(a+b x)^2}}\right)}{\left(-1+2 a^2\right)\,b^2 x}\right]}{a^2\,\left(1-a^2\right)^{3/2}}\right)$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCsc}[a+b x]}{x^4} \, dx$$

Optimal (type 3, 180 leaves, 9 steps):

$$-\frac{b\,(a+b x)\,\sqrt{1-\frac{1}{(a+b x)^2}}}{6 a\,\left(1-a^2\right)\,x^2}+\frac{\left(2-5 a^2\right)\,b^2\,(a+b x)\,\sqrt{1-\frac{1}{(a+b x)^2}}}{6 a^2\,\left(1-a^2\right)^2\,x}-\frac{b^3\,\text{ArcCsc}[a+b x]}{3 a^3}-\frac{\text{ArcCsc}[a+b x]}{3 x^3}-\frac{\left(2-5 a^2+6 a^4\right)\,b^3\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b x]\right]}{\sqrt{1-a^2}}\right]}{3 a^3\,\left(1-a^2\right)^{5/2}}$$

Result (type 3, 241 leaves):

$$\frac{1}{6}\left(\frac{b\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\,\left(a^4+a\,b\,x-4\,a^3\,b\,x+2\,b^2\,x^2-a^2\,\left(1+5\,b^2\,x^2\right)\right)}{a^2\,\left(-1+a^2\right)^2\,x^2}-\right.$$

$$\left.\frac{2\,\text{ArcCsc}[a+b\,x]}{x^3}-\frac{2\,b^3\,\text{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^3}+\frac{i\,\left(2-5\,a^2+6\,a^4\right)\,b^3\,\text{Log}\left[\frac{12\,a^3\,\left(-1+a^2\right)^2\,\left(-\frac{i\,\left(-1+a^2+a\,b\,x\right)}{\sqrt{1-a^2}}-(a+b\,x)\,\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{\left(2-5\,a^2+6\,a^4\right)\,b^3\,x}\right]}{a^3\,\left(1-a^2\right)^{5/2}}\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcCsc}[a+b\,x]}{x^5}\,dx$$

Optimal (type 3, 239 leaves, 10 steps):

$$-\frac{b\,(a+b\,x)\,\sqrt{1-\frac{1}{(a+b\,x)^2}}}{12\,a\,\left(1-a^2\right)\,x^3}+\frac{\left(3-8\,a^2\right)\,b^2\,(a+b\,x)\,\sqrt{1-\frac{1}{(a+b\,x)^2}}}{24\,a^2\,\left(1-a^2\right)^2\,x^2}-\frac{\left(6-17\,a^2+26\,a^4\right)\,b^3\,(a+b\,x)\,\sqrt{1-\frac{1}{(a+b\,x)^2}}}{24\,a^3\,\left(1-a^2\right)^3\,x}+$$

$$\frac{b^4\,\text{ArcCsc}[a+b\,x]}{4\,a^4}-\frac{\text{ArcCsc}[a+b\,x]}{4\,x^4}+\frac{\left(2-7\,a^2+8\,a^4-8\,a^6\right)\,b^4\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\,\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{4\,a^4\,\left(1-a^2\right)^{7/2}}$$

Result (type 3, 307 leaves):

$$\frac{1}{8}\left(\frac{1}{3\,a^3\,\left(-1+a^2\right)^3\,x^3}\right.$$

$$b\,\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\,\left(2\,a^7-6\,a^6\,b\,x+3\,a\,b^2\,x^2+6\,b^3\,x^3+a^3\,\left(2-6\,b^2\,x^2\right)+2\,a^5\,\left(-2+9\,b^2\,x^2\right)+a^4\,b\,x\,\left(7+26\,b^2\,x^2\right)-a^2\,\left(b\,x+17\,b^3\,x^3\right)\right)-$$

$$\left.\frac{2\,\mathrm{ArcCsc}[a+b\,x]}{x^4}+\frac{2\,b^4\,\mathrm{ArcSin}\left[\frac{1}{a+b\,x}\right]}{a^4}+\frac{i\,\left(-2+7\,a^2-8\,a^4+8\,a^6\right)\,b^4\,\mathrm{Log}\left[\frac{16\,a^4\,\left(-1+a^2\right)^3\,\left(\frac{i\,\left(-1+a^2+a\,b\,x\right)}{\sqrt{1-a^2}}+(a+b\,x)\,\sqrt{\frac{-1+a^2+2\,a\,b\,x+b^2\,x^2}{(a+b\,x)^2}}\right)}{\left(-2+7\,a^2-8\,a^4+8\,a^6\right)\,b^4\,x}\right]}{a^4\,\left(1-a^2\right)^{7/2}}\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{\mathrm{ArcCsc}[a+b\,x]^2}{x}\,dx$$

Optimal (type 4, 324 leaves, 17 steps):

$$\mathrm{ArcCsc}[a+b\,x]^2\,\mathrm{Log}\left[1+\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+\mathrm{ArcCsc}[a+b\,x]^2\,\mathrm{Log}\left[1+\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\mathrm{ArcCsc}[a+b\,x]^2\,\mathrm{Log}\left[1-e^{2\,i\,\mathrm{ArcCsc}[a+b\,x]}\right]-$$

$$2\,i\,\mathrm{ArcCsc}[a+b\,x]\,\mathrm{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]-2\,i\,\mathrm{ArcCsc}[a+b\,x]\,\mathrm{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]+$$

$$i\,\mathrm{ArcCsc}[a+b\,x]\,\mathrm{PolyLog}\left[2,e^{2\,i\,\mathrm{ArcCsc}[a+b\,x]}\right]+2\,\mathrm{PolyLog}\left[3,-\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]+2\,\mathrm{PolyLog}\left[3,-\frac{i\,a\,e^{i\,\mathrm{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]-\frac{1}{2}\,\mathrm{PolyLog}\left[3,e^{2\,i\,\mathrm{ArcCsc}[a+b\,x]}\right]$$

Result (type 4, 1217 leaves):

$$\frac{i\,\pi^3}{6}-\frac{1}{3}\,i\,\mathrm{ArcCsc}[a+b\,x]^3+8\,i\,\mathrm{ArcCsc}[a+b\,x]\,\mathrm{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\mathrm{ArcTan}\left[\frac{(1+a)\,\mathrm{Cot}\left[\frac{1}{4}\,(\pi+2\,\mathrm{ArcCsc}[a+b\,x])\right]}{\sqrt{1-a^2}}\right]-$$

$$8\,i\,\mathrm{ArcCsc}[a+b\,x]\,\mathrm{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\,\mathrm{ArcTan}\left[\frac{(1+a)\,\left(\mathrm{Cos}\left[\frac{1}{2}\,\mathrm{ArcCsc}[a+b\,x]\right]-\mathrm{Sin}\left[\frac{1}{2}\,\mathrm{ArcCsc}[a+b\,x]\right]\right)}{\sqrt{1-a^2}\,\left(\mathrm{Cos}\left[\frac{1}{2}\,\mathrm{ArcCsc}[a+b\,x]\right]+\mathrm{Sin}\left[\frac{1}{2}\,\mathrm{ArcCsc}[a+b\,x]\right]\right)}\right]-$$

$$\text{ArcCsc}[a+b x]^2 \,\text{Log}\left[1-e^{-i\,\text{ArcCsc}[a+b x]}\right]-\pi\,\text{ArcCsc}[a+b x]\,\text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]+$$

$$\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]+4\,\text{ArcCsc}[a+b x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]-$$

$$\pi\,\text{ArcCsc}[a+b x]\,\text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]+\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]-$$

$$4\,\text{ArcCsc}[a+b x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(1+\sqrt{1-a^2}\right)e^{-i\,\text{ArcCsc}[a+b x]}}{a}\right]-\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+e^{i\,\text{ArcCsc}[a+b x]}\right]+$$

$$\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{-i+\sqrt{-1+a^2}}\right]+\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1-\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{i+\sqrt{-1+a^2}}\right]+$$

$$\pi\,\text{ArcCsc}[a+b x]\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$4\,\text{ArcCsc}[a+b x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\left(-1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]+$$

$$\pi\,\text{ArcCsc}[a+b x]\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-\text{ArcCsc}[a+b x]^2\,\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]+$$

$$4\,\text{ArcCsc}[a+b x]\,\text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{\left(1+\sqrt{1-a^2}\right)\left(\frac{1}{a+b x}+i\sqrt{1-\frac{1}{(a+b x)^2}}\right)}{a}\right]-$$

$$2\,i\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,e^{-i\,\text{ArcCsc}[a+b x]}\right]+2\,i\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,-e^{i\,\text{ArcCsc}[a+b x]}\right]-$$

$$2\,i\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,-\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{-i+\sqrt{-1+a^2}}\right]-2\,i\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[2,\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{i+\sqrt{-1+a^2}}\right]-$$

$$2\,\text{PolyLog}\left[3,e^{-i\,\text{ArcCsc}[a+b x]}\right]-2\,\text{PolyLog}\left[3,-e^{i\,\text{ArcCsc}[a+b x]}\right]+2\,\text{PolyLog}\left[3,-\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{-i+\sqrt{-1+a^2}}\right]+2\,\text{PolyLog}\left[3,\frac{a\,e^{i\,\text{ArcCsc}[a+b x]}}{i+\sqrt{-1+a^2}}\right]$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCsc}[a+b\,x]^2}{x^2}\,dx$$

Optimal (type 4, 254 leaves, 12 steps):

$$-\frac{b\,\text{ArcCsc}[a+b\,x]^2}{a}-\frac{\text{ArcCsc}[a+b\,x]^2}{x}-\frac{2\,i\,b\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}+$$
$$\frac{2\,i\,b\,\text{ArcCsc}[a+b\,x]\,\text{Log}\left[1+\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}-\frac{2\,b\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1-\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}+\frac{2\,b\,\text{PolyLog}\left[2,-\frac{i\,a\,e^{i\,\text{ArcCsc}[a+b\,x]}}{1+\sqrt{1-a^2}}\right]}{a\,\sqrt{1-a^2}}$$

Result (type 4, 804 leaves):

$$-\frac{1}{a}b\left(\frac{(a+b\,x)\,\text{ArcCsc}[a+b\,x]^2}{b\,x}+\frac{2\,\pi\,\text{ArcTan}\left[\frac{a-\text{Tan}\left[\frac{1}{2}\text{ArcCsc}[a+b\,x]\right]}{\sqrt{1-a^2}}\right]}{\sqrt{1-a^2}}+\right.$$

$$\frac{1}{\sqrt{-1+a^2}}2\left(-2\,\text{ArcCos}\left[\frac{1}{a}\right]\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]+(\pi-2\,\text{ArcCsc}[a+b\,x])\right.$$

$$\text{ArcTanh}\left[\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]+\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\left(-\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]+\right.\right.$$

$$\left.\left.\text{ArcTanh}\left[\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]\right)\right)\text{Log}\left[\frac{\sqrt{-1+a^2}\;e^{\frac{1}{4}i\,(\pi-2\,\text{ArcCsc}[a+b\,x])}}{\sqrt{2}\,\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]+$$

$$\left(\text{ArcCos}\left[\frac{1}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]-2\,i\,\text{ArcTanh}\left[\frac{(-1+a)\,\text{Tan}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{-1+a^2}\;e^{\frac{1}{2}i\,\text{ArcCsc}[a+b\,x]}}{\sqrt{a}\,\sqrt{-\frac{b\,x}{a+b\,x}}}\right]-\left(\text{ArcCos}\left[\frac{1}{a}\right]-2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b\,x])\right]}{\sqrt{-1+a^2}}\right]\right)$$

$$\text{Log}\left[\frac{(-1+a)\left(i+i a+\sqrt{-1+a^2}\right)\left(-i+\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}{a\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}\right]-\left(\text{ArcCos}\left[\frac{1}{a}\right]+\right.$$

$$\left.2\,i\,\text{ArcTanh}\left[\frac{(1+a)\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]}{\sqrt{-1+a^2}}\right]\right)\text{Log}\left[\frac{(-1+a)\left(-i-i a+\sqrt{-1+a^2}\right)\left(i+\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}{a\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}\right]+$$

$$i\left(-\text{PolyLog}\left[2,\frac{\left(1-i\sqrt{-1+a^2}\right)\left(1-a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}{a\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}\right]+\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(1+i\sqrt{-1+a^2}\right)\left(1-a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}{a\left(-1+a+\sqrt{-1+a^2}\,\text{Cot}\left[\frac{1}{4}(\pi+2\,\text{ArcCsc}[a+b x])\right]\right)}\right]\right)\right)\right)$$

Problem 36: Unable to integrate problem.

$$\int\frac{\text{ArcCsc}[a+b x]^3}{x}\,dx$$

Optimal (type 4, 448 leaves, 20 steps):

$$\text{ArcCsc}[a+b x]^3\,\text{Log}\left[1+\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]+\text{ArcCsc}[a+b x]^3\,\text{Log}\left[1+\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\text{ArcCsc}[a+b x]^3\,\text{Log}\left[1-e^{2 i\,\text{ArcCsc}[a+b x]}\right]-$$

$$3 i\,\text{ArcCsc}[a+b x]^2\,\text{PolyLog}\left[2,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]-3 i\,\text{ArcCsc}[a+b x]^2\,\text{PolyLog}\left[2,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]+$$

$$\frac{3}{2} i\,\text{ArcCsc}[a+b x]^2\,\text{PolyLog}\left[2,e^{2 i\,\text{ArcCsc}[a+b x]}\right]+6\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[3,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]+$$

$$6\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[3,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{2}\,\text{ArcCsc}[a+b x]\,\text{PolyLog}\left[3,e^{2 i\,\text{ArcCsc}[a+b x]}\right]+$$

$$6 i\,\text{PolyLog}\left[4,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]+6 i\,\text{PolyLog}\left[4,-\frac{i a\,e^{i\,\text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]-\frac{3}{4} i\,\text{PolyLog}\left[4,e^{2 i\,\text{ArcCsc}[a+b x]}\right]$$

Result (type 8, 14 leaves):

$$\int\frac{\text{ArcCsc}[a+b x]^3}{x}\,dx$$

Problem 37: Unable to integrate problem.

$$\int \frac{\text{ArcCsc}[a+b x]^3}{x^2} \, dx$$

Optimal (type 4, 378 leaves, 14 steps):

$$-\frac{b \, \text{ArcCsc}[a+b x]^3}{a}-\frac{\text{ArcCsc}[a+b x]^3}{x}-\frac{3 \, i \, b \, \text{ArcCsc}[a+b x]^2 \, \text{Log}\left[1+\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+$$
$$\frac{3 \, i \, b \, \text{ArcCsc}[a+b x]^2 \, \text{Log}\left[1+\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}-\frac{6 \, b \, \text{ArcCsc}[a+b x] \, \text{PolyLog}\left[2,-\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+$$
$$\frac{6 \, b \, \text{ArcCsc}[a+b x] \, \text{PolyLog}\left[2,-\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}-\frac{6 \, i \, b \, \text{PolyLog}\left[3,-\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1-\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}+\frac{6 \, i \, b \, \text{PolyLog}\left[3,-\frac{i \, a \, e^{i \, \text{ArcCsc}[a+b x]}}{1+\sqrt{1-a^2}}\right]}{a \sqrt{1-a^2}}$$

Result (type 8, 14 leaves):

$$\int \frac{\text{ArcCsc}[a+b x]^3}{x^2} \, dx$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int x^3 \, \text{ArcCsc}\left[a+b x^4\right] \, dx$$

Optimal (type 3, 48 leaves, 6 steps):

$$\frac{\left(a+b x^4\right) \text{ArcCsc}\left[a+b x^4\right]}{4 b}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{\left(a+b x^4\right)^2}}\right]}{4 b}$$

Result (type 3, 127 leaves):

$$\frac{\left(a+b x^4\right) \text{ArcCsc}\left[a+b x^4\right]}{4 b}+\frac{\sqrt{-1+\left(a+b x^4\right)^2} \left(-\text{Log}\left[1-\frac{a+b x^4}{\sqrt{-1+\left(a+b x^4\right)^2}}\right]+\text{Log}\left[1+\frac{a+b x^4}{\sqrt{-1+\left(a+b x^4\right)^2}}\right]\right)}{8 b \left(a+b x^4\right) \sqrt{1-\frac{1}{\left(a+b x^4\right)^2}}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int x^{-1+n}\,\text{ArcCsc}\left[a+b\,x^n\right]\,dx$$

Optimal (type 3, 48 leaves, 6 steps):

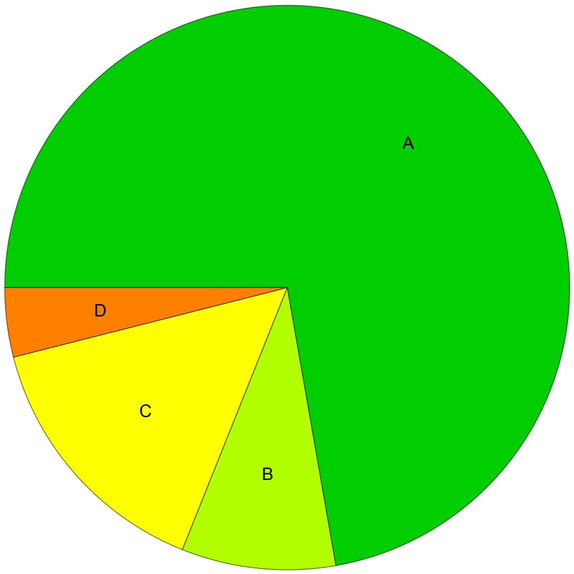
$$\frac{\left(a+b\,x^n\right)\,\text{ArcCsc}\left[a+b\,x^n\right]}{b\,n}+\frac{\text{ArcTanh}\left[\sqrt{1-\frac{1}{\left(a+b\,x^n\right)^2}}\right]}{b\,n}$$

Result (type 3, 130 leaves):

$$\frac{\left(a+b\,x^n\right)\,\text{ArcCsc}\left[a+b\,x^n\right]}{b\,n}+\frac{\sqrt{-1+\left(a+b\,x^n\right)^2}\,\left(-\text{Log}\left[1-\frac{a+b\,x^n}{\sqrt{-1+\left(a+b\,x^n\right)^2}}\right]+\text{Log}\left[1+\frac{a+b\,x^n}{\sqrt{-1+\left(a+b\,x^n\right)^2}}\right]\right)}{2\,b\,n\,\left(a+b\,x^n\right)\,\sqrt{1-\frac{1}{\left(a+b\,x^n\right)^2}}}$$

Summary of Integration Test Results

227 integration problems



A - 164 optimal antiderivatives

B - 20 more than twice size of optimal antiderivatives

C - 34 unnecessarily complex antiderivatives

D - 9 unable to integrate problems

E - 0 integration timeouts